

MDAG.com Internet Case Study 64

MDAG-com Case Study 64 - Minesite Components as Large Physical Analogues of First-Order Low-Pass Signal Filters, Explaining 1-over-f and Other Fractal Spectral Slopes for Flows and Chemistries

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To repeat two important quotations from MDAG Case Study 63 (Morin 2020):

It has been known for over two centuries that the solid and liquid Earth and its atmosphere are almost permanently electrified. (National Research Council, 1986)

EMFs (electrical and magnetic fields) are all around us, occurring naturally in every atom of matter. The surface of the Earth is covered with a natural electric field, created by electrical charges in the upper atmosphere. Similarly, a powerful electric field is required to keep the cells of living organisms alive. (Hydro Québec, 2011)

Abstract

In this MDAG Case Study, two major questions are asked and answered. These offer explanations for many currently unexplained observations in the periodicities of minesite-drainage flows and chemistries. This includes causes of periodicities at many wavelengths and of fractal spectral slopes including “1-over-f” slopes.

- 1) What mechanisms and processes can introduce periodicity into minesite-drainage chemistry and flow? In other words, why do flow and chemistry “pulsate” at many wavelengths? This effect is more “ubiquitous” in non-mining catchments.

Answer: Based on Equation 2-2 reflecting various forms of energy and their relationships, the oscillation in any energy term can produce periodicity in one or more other energy terms. This includes the 11-yr sunspot cycle, the 2.38-yr quasi-biennial oscillation in tropical zonal winds, rotational gravity waves, and various tides driven by motions of the moon and sun relative to the earth. These appear to be linked to flow and aqueous concentrations of minesite drainage primarily through oscillations in large-scale electrical fields based on electrodynamics, electrochemistry, etc. This was explained in the previous MDAG Case Study 63.

2) How do these mechanisms apparently “self organize” so that their amplitudes and spectral powers generally decrease with decreasing wavelength, often forming fractal spectral slopes? This includes the very common yet mysterious “1-over-f” with a spectral fractal slope of 1.0, which should not appear and persist according to mathematics.

Answer: Minesite components are open systems in the surficial environment, well grounded to the earth that behaves like a capacitor. Thus, relatively large components can act as first-order low-pass filters. These filters cause the spectral powers of various periodicities entering them to (1) decrease along a fractal slope of 2 at wavelengths shorter than the “cutoff wavelength” and (2) remain unfiltered at longer wavelengths. When several mechanisms are simultaneously acting as low-pass filters, 1-over-f slopes can appear.

If these answers withstand scrutiny, then fractal spectral slopes including 1-over-f slopes should be common in drainages from relatively large minesite components, and they are, and thus not mysterious and unexplainable.

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1. Introduction

Where minesite-drainage monitoring has been carried out at sufficiently high sampling frequencies and durations, interesting and intriguing patterns have been seen in the time series. The basic observation is that flow rates and aqueous concentrations of minesite drainages are not simple or steady, at least not in these rare cases with long-term high-frequency sampling. They are not stochastic, but also not deterministic (Morin, 2018b). They are not random or chaotic (Morin, 2019b). They display periodicity in complex ways. Morin and Hutt (2001) pessimistically emphasized,

“this regularity reflects the summation of the effects of all other (countless) physical, chemical, and biological processes operating at a minesite . . . It may involve so many processes that the explanation may never be known.”

For about 200 years, spectral analysis has been a mathematical tool that illustrates peak spectral powers at various wavelengths, where spectral power is proportional to the square of the amplitude. For minesite drainages as well as non-mining-related rivers and catchments, the observed overall spectral trend typically is decreasing spectral power (\sim amplitude²) of the peaks with decreasing wavelength, with the plot often called a “periodogram”. The resulting slopes in many periodograms are commonly fractal, ranging from zero (random) to 2 (random walk) with the slope of 1 (“1-over-f slope”) being the most complex (Morin, 2019a and b; Morin, 2018a and b; Morin, 2016). Some examples are shown in Figures 1-1 to 1-5 below.

Remarkably, 1-over-f slopes have been documented in many sciences and arts. Here is a partial list of their occurrence: earthquakes described through the Gutenberg–Richter law, climatic temperature and precipitation, highway traffic flow, river flow, tides, heart beats, neural activity, biologic evolution, solar flares, psychological models of mental states, light from quasars, electrical current in solid-state devices, epidemics, variations in musical styles, insulin uptake by diabetics, economic trends, forest fires, application of automotive paint, and cavitation in pumps. Therefore, fractal 1-over-f slopes are common, but no comprehensive explanation has been yet found for them.

To examine sources of periodicity, the previous MDAG Case Study 63 (Morin, 2020) focussed on electrical fields. This is because small changes in ground electrical potential, considered minor background electrical “noise” by some, can significantly affect aqueous chemistry. For example, a small change in Eh of only 18 mV and in pH of only 0.10 units (ideally equivalent to a change of 6 mV) can cause factor-of-two changes in aqueous iron concentrations.

Electrical fields in the Earth are inevitably linked to other processes like large and small physical movements, magnetic variations in the earth, weather systems, and cosmic radiation. For example, the simple physical movement of water through a porous or fractured medium can create an electrical field that in turn affects the distribution of ions in that water (“electrohydrodynamics”).

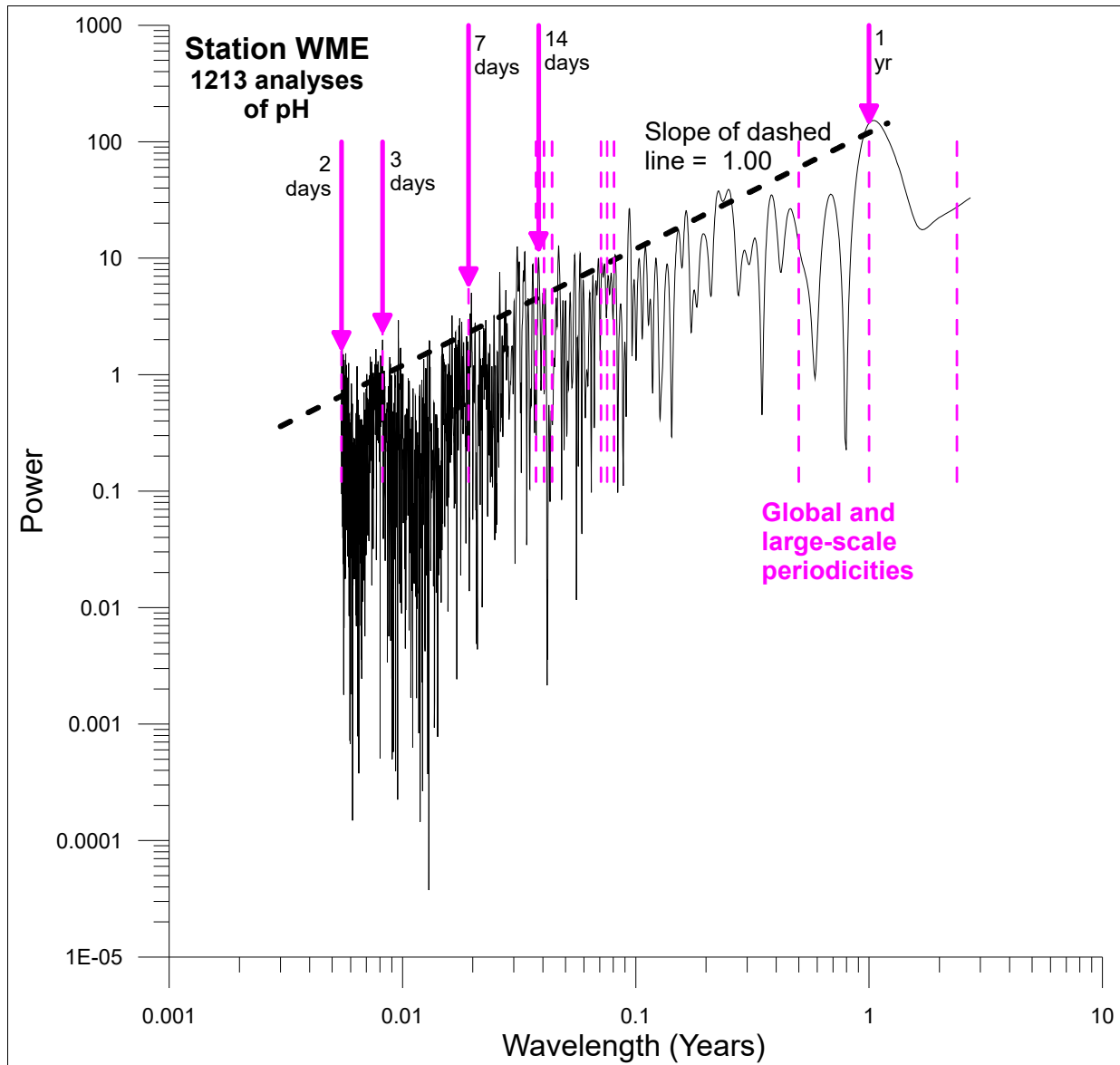


Figure 1-1. Spectral periodogram for pH at a downstream station at Minesite 1 (from Morin, 2016 and 2018a), with potentially significant wavelengths of large-scale periodicities added as vertical coloured dashed lines, and vertical downward-pointing arrows highlighting some significant wavelengths (from Morin, 2020); see also Figure 1-2.

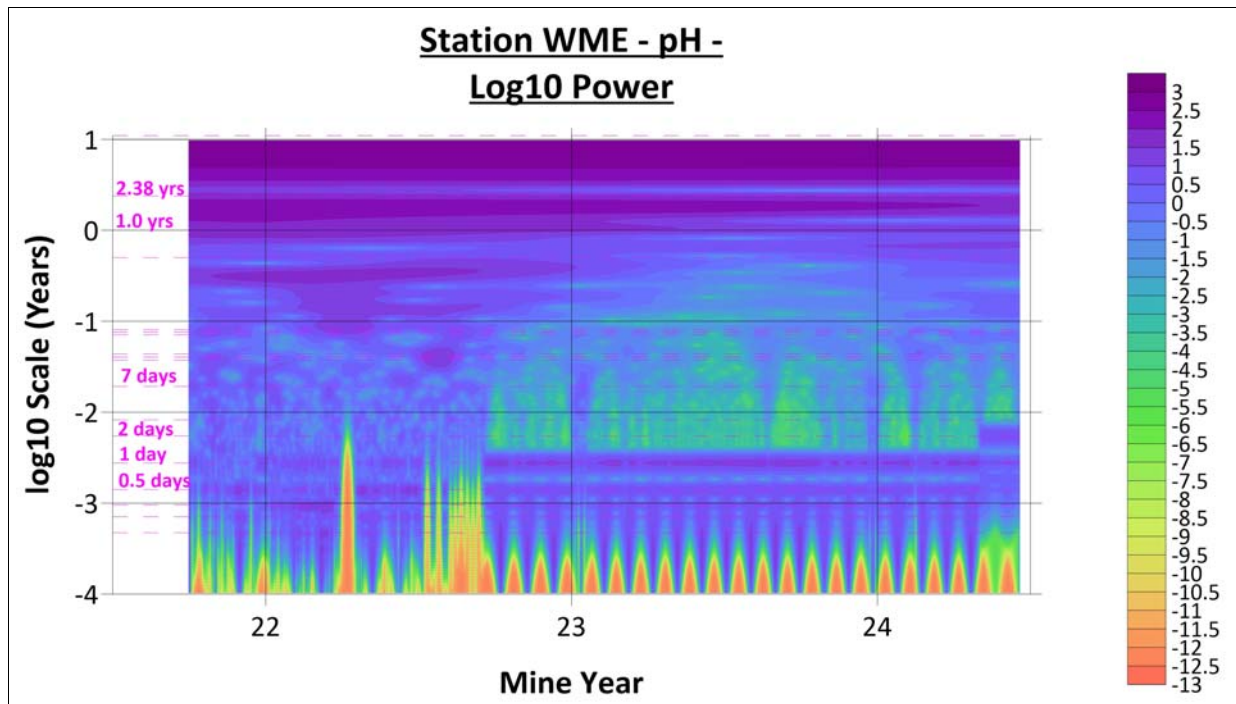


Figure 1-2. Wavelet transform of wavelet power for pH at a downstream station at Minesite 1 (from Morin, 2016 and 2018a), with potentially significant wavelengths of large-scale periodicities added as horizontal coloured dashed lines (from Morin, 2020); this is the same data as in Figure 1-1, but wavelet transforms can highlight short-term periodicities at shorter wavelengths that spectral periodograms cannot detect.

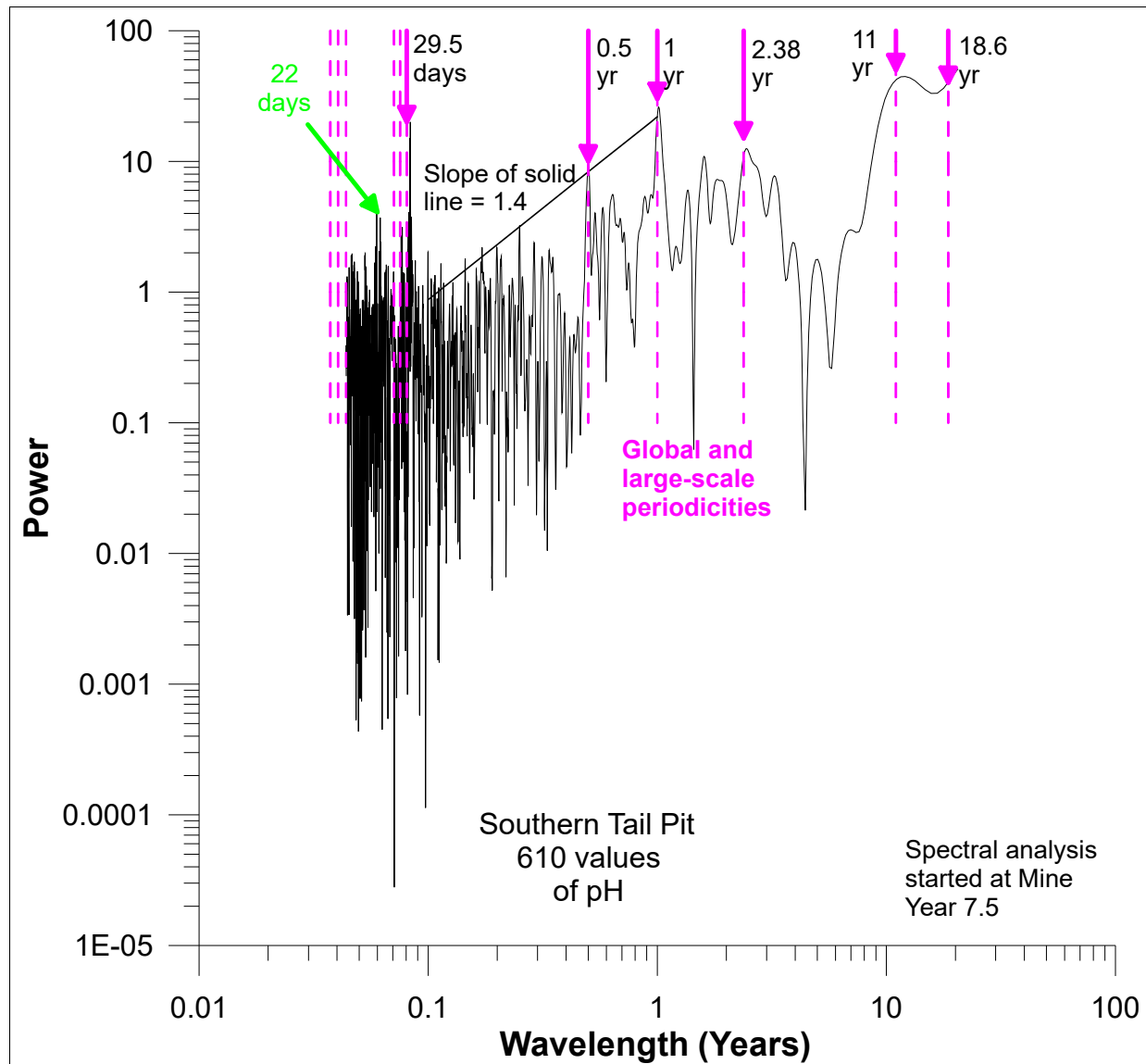


Figure 1-3. Spectral periodogram for pH in flow from a waste-rock-backfilled pit at Minesite 2 (Morin 2016 and 2018a), with potentially significant wavelengths of large-scale periodicities added as vertical coloured dashed lines (Morin, 2020).

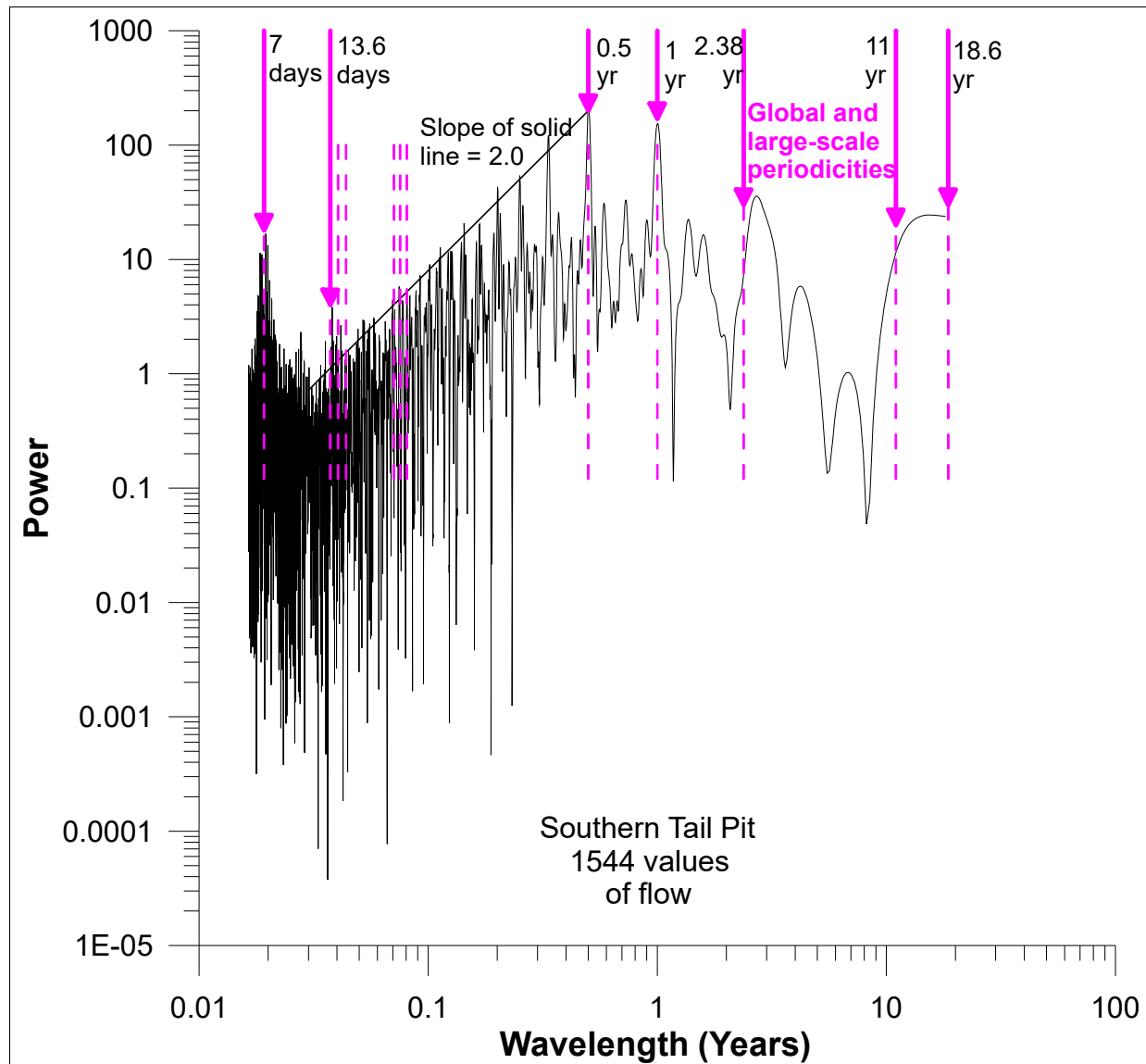


Figure 1-4. Spectral periodogram for flow rate from a waste-rock-backfilled pit at Minesite 2 (Morin, 2016 and 2018a), with potentially significant wavelengths of large-scale periodicities added as vertical coloured dashed lines (Morin, 2020).

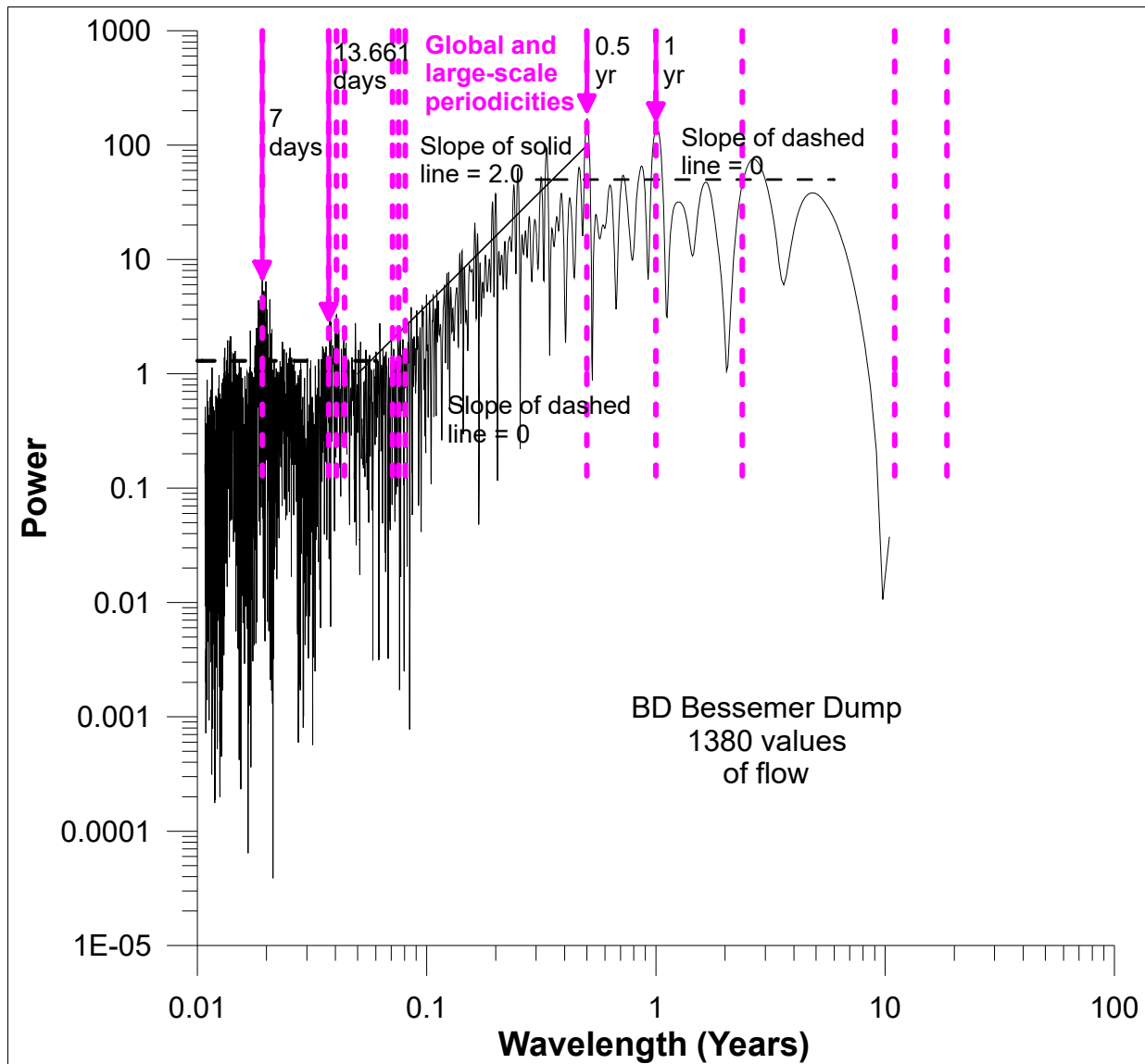


Figure 1-5. Spectral periodogram for flow rate at the toe of the large waste-rock pile at Minesite 2 (Morin, 2016 and 2018a), with potentially significant wavelengths of large-scale periodicities added as vertical coloured dashed lines (Morin, 2020).

As a result, planetary and large-scale cycles and tides can introduce periodicities of various wavelengths into flows and chemistries at minesites (Morin, 2020), as illustrated in Figures 1-1 to 1-5 above. Smaller-scale, site-specific cycles, such as in local temperature and barometric pressure, can add other significant periodicities. This discussion is expanded in this case study.

Identifying the sources of some wavelengths of periodicity is not sufficient to explain the fractal slopes common in periodograms created by these periodicities. For example, amplitudes of short-term periodicities like ocean tides can be high, but their spectral power is suppressed in minesite-drainage periodograms. Therefore, this MDAG case study focusses on two issues.

- 1) What mechanisms and processes can introduce periodicity into minesite-drainage chemistry and flow? In other words, why do flow and chemistry “pulsate” at many wavelengths? This effect is more “ubiquitous” in non-mining catchments.

- 2) How do these mechanisms apparently “self organize” so that their amplitudes and spectral powers generally decrease with decreasing wavelength, often forming fractal spectral slopes? This includes the very common yet mysterious “1-over-f” with a spectral fractal slope of 1.0, which should not appear and persist according to mathematics.

2. What other mechanisms and processes can introduce periodicity into minesite-drainage chemistry and flow?

2.1 The “Big Picture” with Dreaded Thermodynamics Side-Stepped

There is a very general, “big picture” way to answer this question. It starts with considering the following scenario. A minesite component experiences the entry, the transit, and the exit of water, other mass (like gases), and various forms of energy (Figure 2-1). The component affects water as water flows through, and in turn is affected by the flow.

Because a flow rate exists and may contain detectable aqueous concentrations, the observed effects of these scenarios can be viewed as a “signal filter” (Figure 2-2). For elements below detection, the effects can be viewed as a “signal generator”. In all cases, the cumulative energy is altered. One may think simplistically that the “chemical energy” has been changed in some way due to changes in absolute concentrations, but reality can be much grander and more complex and more fascinating than that.

For either scenario above, we can define a “total” amount of energy in a minesite component that changes with time because it is an “open system” on the earth’s surface. Numerous “forms” of energy, or if preferred numerous “carriers” of energy (Falk et al., 1983), and matter can enter or leave it through time and distance.

At this point, the dreaded and fearful topic of thermodynamics briefly appears. However, it will be conveniently side-stepped here by focussing on generally functional equations for total energy in open systems that might not meet the expectations of thermodynamic purists. For example, a recent re-definition of entropy makes sense to me (Rüffler and Job, 2011; Job and Rüffler, 2016; Wu and Wu, 2020), but there has been backlash against it. For example, there can be some confusion between heat, energy driven by a thermal gradient, and entropy (“unavailable” energy).

Also, thermodynamics often implies equilibrium, at least at some scale (in “non-equilibrium” thermodynamics), and is based on some constant conditions like temperature or pressure. None of these restrictions are included here in these open systems that can be far from classical equilibrium. My apologies to thermodynamics!

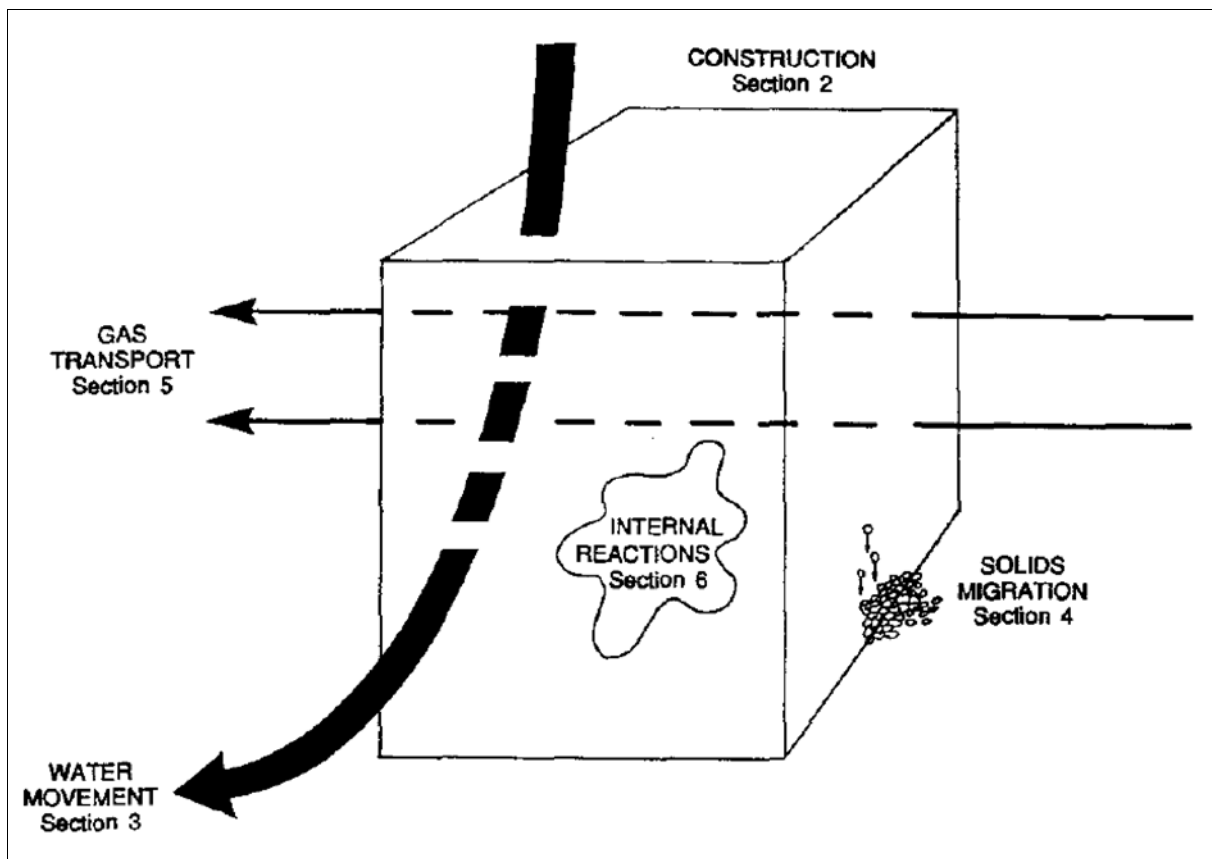


Figure 2-1. A waste-rock pile as a simple conceptual system (from Morin et al., 1991).

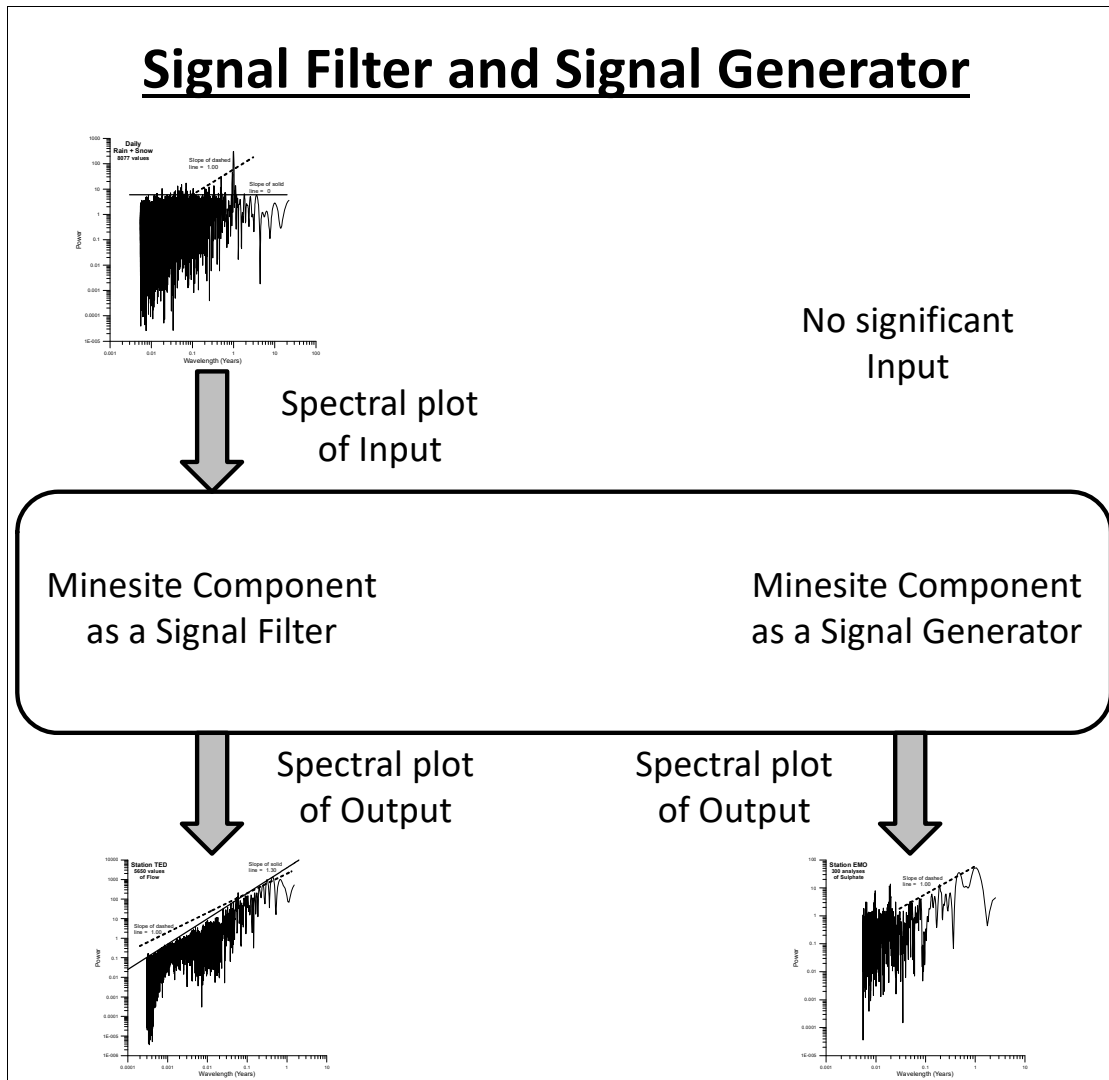


Figure 2-2. Schematic diagram of a minesite component acting as a signal filter, changing the spectral plot of the input to that of the output, and as a signal generator, producing a spectral plot in output with no significant input (from Morin, 2016).

2.2 A Generic Energy Balance for Open Environmental Systems

A functional equation for total energy of the minesite component as an open system is:

$$E_t = E_{un} + E_{av} = TS + E_{av} \quad (\text{Equation 2-1})$$

where:

E_t = total energy, in joules

E_{un} = “unavailable” energy that is “wasted” or “locked away” that “cannot do anything”, in joules,
and $dE_{un} > 0$

E_{av} = “available” energy that can “do something”, in joules

We are interested in E_{av} . However, we will retain E_{un} to be more complete.

E_{un} and E_{av} in Equation 2-1 can be partly expanded with many terms (e.g., Wikipedia, 2020a).

$$E_t = TS + Vp + \sum \mu_i N_i + q\phi + m\Psi + \frac{1}{2} mv^2 + \dots \quad (\text{Equation 2-2})$$

where:

E_t = total energy, in joules

T = temperature, in Kelvin

S = entropy, in joules/K

V = volume, in meter³

p = pressure, in pascal (joules/meter³)

μ_i = chemical potential of the i th species, in joules/mole

N_i = number of moles (particles) of the i th species, in moles

ϕ = electrical potential in volts

q = electrical charge, in coulombs

Ψ = gravitation potential, in joules/kg

m = mass, in kg

v = velocity, in meter/second

(note: positive and negative signs can sometimes shift with parameter and methodology)

Other terms can be added to Equation 2-2, such as radiation and biological energy. Therefore, Equation 2-2 is not exhaustive and complete.

Equation 2-2 does not say anything about variation or periodicity, because it describes energy “content”. In reality, the energy of open systems like minesite components can change with time and/or vary with spatial location. Thus, sequential small changes in the left-side term and the right-side terms of Equation 2-2 are important (e.g., dE_t = small change in cumulative total energy, in joules).

For example, ignoring most of the terms on the right side, or assuming most variables remain constant, leads to:

$$dE_t = Vdp - SdT + \sum \mu_i dN_i \quad (\text{Equation 2-3})$$

Equation 2-3 is one form of Gibbs free energy (Wikipedia, 2020a), with $dG = dE_t$. Please note all

the missing or assumed constant variables that highlight a limitation of Gibbs free energy and other free energies like Helmholtz.

Thermodynamics reveals in many terms on the right side of Equation 2-2. Each of those terms consists of a mathematical product of two “conjugate quantities” (Wikipedia, 2020b and 2020c). One is often considered an “intensive” property of the system (i.e., it does not depend on quantity or size of the system) and the other an “extensive” property (i.e., it varies with the size of the system).

As a result, only one variable in each term should vary (with a “d” in front of it in this case study). The problem is a clear demarcation of intensive and extensive variables is not often possible (Wikipedia, 2020b; Redlich, 1969). So this case study is free to place a “d” in front of most variables as needed, which is seen below.

To show the comprehensive value of Equation 2-2, the following two subsections (2.3 and 2.4) illustrate other simplifications of Equation 2-2 that provide the common equations for physical groundwater flow and aqueous geochemistry. Both are important aspects of minesite drainage, but each is limited to only certain terms in Equation 2-2.

2.3 Simplifications for Physical Hydrogeology

In the field of physical hydrogeology, the flow of groundwater is caused by a decrease in energy along flowpaths. With water assumed virtually incompressible ($dV = 0$), this is often simplified to:

$$dE_t = Vdp + md\Psi \quad (\text{Equation 2-4})$$

where dE_t is based on only two terms taken from Equation 2-2.

For the second term on the right, the gravitational constant (g) is often considered constant and separated from the differential to produce:

$$dE_t = Vdp + mgdz \quad (\text{Equation 2-5})$$

where:

- g = gravitational acceleration, or standard gravity, in meter/second²
- z = elevation, in meters

For simplicity, these terms are divided by the weight of incompressible water to obtain hydraulic heads measurable as elevations of water in piezometers:

$$dh = dv + dz \quad (\text{Equation 2-6})$$

where:

- h = hydraulic or piezometric head, in meters, measured as the elevation of groundwater level in a piezometer
- v = pressure head, in meters, measured as the height of the water column inside a piezometer
- z = elevation, in meters, of the piezometer intake screen.

For water moving at significant velocities, more typical of flowing surface water, the effect of changing velocity ($\frac{1}{2} \rho v^2$) can be included on the right side of Equation 2-5. When divided by the weight of incompressible water, the resulting velocity head can be added to the right side of Equation 2-6.

It is interesting to track Equation 2-6 backwards to Equation 2-2 to see how much energy may be ignored in this standard hydrogeologic approach. This could lead to errors if the terms missing from the right side of Equation 2-2 change significantly.

2.4 Simplifications for Aqueous Geochemistry and Electrochemistry

In the field of aqueous geochemistry, particularly minesite-drainage chemistry, Equation 2-2 can be simplified for a homogenous system to:

$$dE_t = \sum \mu_i dN_i \quad (\text{Equation 2-7})$$

where dE_t is based on only one term taken from Equation 2-2. In this case, dE_t is identical to the change in Gibbs free energy in homogeneous systems (dG , Wikipedia, 2020a), which is minimized at chemical equilibrium. All the missing terms from Equation 2-2 highlight why chemical equilibrium should not necessarily be expected in the open systems of minesite drainage.

Section 3 of Morin (2020) highlighted some of the ambiguity in the definitions of total chemical potential and electrochemical potential. Equation 2-2 avoids that ambiguity by giving each form of energy, potentially moved by a force or gradient in the corresponding conjugate quantity, its own term on the right side.

Section 4 of Morin (2020) highlighted why ionic charge balances in water do not necessarily have to be zero. Equation 2-2 shows that the change in one energy term (e.g., chemical energy) can be partially or fully offset by the change in other terms and/or a change in total energy, so zero values for ion balances are not required.

Morin (2020) and Section 1 above of this case study explained how a small change in electrical potential on the order of 10 mV could change an aqueous concentration by a factor of two. Equation 2-2 now shows a hidden assumption in this statement:

$$dE_t = \sum \mu_i dN_i + qd\phi, \text{ with } dE_t = \text{constant or } dE_t \neq qd\phi \quad (\text{Equation 2-8})$$

As a result, a change in voltage ($d\phi$) forces a proportional but opposite change in aqueous concentration (dN_i). Reality can be much more complex, where the change in any term in Equation 2-2 can cause aqueous concentrations to vary.

2.5 Causes of Periodicity in Flows and Aqueous Concentrations

Finally, the answer can now be succinctly given to the question: what other mechanisms and processes can introduce periodicity into minesite-drainage chemistry and flow?

The answer is: Equation 2-2 shows that periodicity in the variation of any one term can induce periodicity in one or more other terms. This can be missed by common simplifications (e.g., Sections 2.3 and 2.4).

Morin (2020) focussed on periodicity in the earth's electrical fields and how this could induce periodicity in minesite-drainage chemistry with Equation 2-8, reflecting topics like electrohydrodynamics. Many studies of minesite drainage, particularly when sulphide minerals are reactive, emphasize the importance of electrochemistry. For example, "Electrochemical processes often play a decisive role . . ." (Kalinnikov et al., 2001).

It is not clear if Equation 2-2 is comprehensive enough to identify most mechanisms contributing to periodicity in minesite-drainage flows and chemistries. If not, then it perhaps fulfills the pessimistic prediction by Morin and Hutt (2001) quoted in Section 1 above.

3. How do these mechanisms apparently “self organize” so that their amplitudes and spectral powers generally decrease with decreasing wavelength, often forming fractal spectral slopes?

Section 1 above, the references in that section, and Figures 1-1 to 1-5 emphasize fractal slopes in spectral periodograms of minesite-drainage flows and chemistries. The potential sources of the multiple wavelengths of periodicity in the periodograms were discussed above in Section 2, with a focus on linking electrical and chemical processes.

However, there is an important issue still not resolved. For example, due to daily tides, a wavelength of one day could produce an amplitude far greater than amplitudes at longer wavelengths (Morin, 2020). Yet, there is some “filter” that causes the amplitude of each shorter wavelength to be generally (not always consistently) less than the preceding longer wavelength.

What is that filter?

An important clue lies in Figures 1-3 to 1-5 above and dozens of figures in the appendices of Morin (2016). This clue is that, above a certain long wavelength such as one year, the fractal filtering stops. Above that “cutoff wavelength”, which can vary by site and chemical element, the peaks of the longer wavelengths form a horizontal line (slope ~ zero) or fluctuate erratically, indicating significant filtering is no longer occurring.

3.1 First-Order Low-Pass Filters

Such behaviour is commonly seen in electronics and is known as a “low-pass filter” (Wikipedia, 2020d). A low-pass filter allows amplitudes at low frequencies to pass through unfiltered, whereas amplitudes at increasing shorter wavelengths are progressively reduced often along a fractal slope. In periodograms like Figures 1-3 to 1-5, a “first-order low-pass filter” will create a spectral slope of 2.0 (e.g., spectral power is reduced by a factor of 4 when wavelength is halved) at shorter wavelengths. Higher order low-pass filters will create steeper slopes. Thus, a slope of 1.0 (1-over-f) often seen in minesite and non-mining drainages is not created by a single low-pass filter.

The action of a first-order low-pass filter was recognized in spectral analysis of minesite drainage at the start (Morin, 2016). The nature of this filter was unknown at that time, but this section will provide a viable explanation.

Figures 3-1 and 3-2 below are taken from Morin (2016) and illustrate various trends that can be seen in first-order low-pass filters. These trends can be seen above in Figures 1-3 to 1-5 including a filtered slope of 2.

Figure 3-3 then illustrates that a complex 1-over-f (slope = 1) can be derived from an overlapping series of first-order low-pass filters at various wavelengths and powers with filtered slopes of 2 (based on the work of Milotti, 2002, and Halley and Inchausti, 2004).

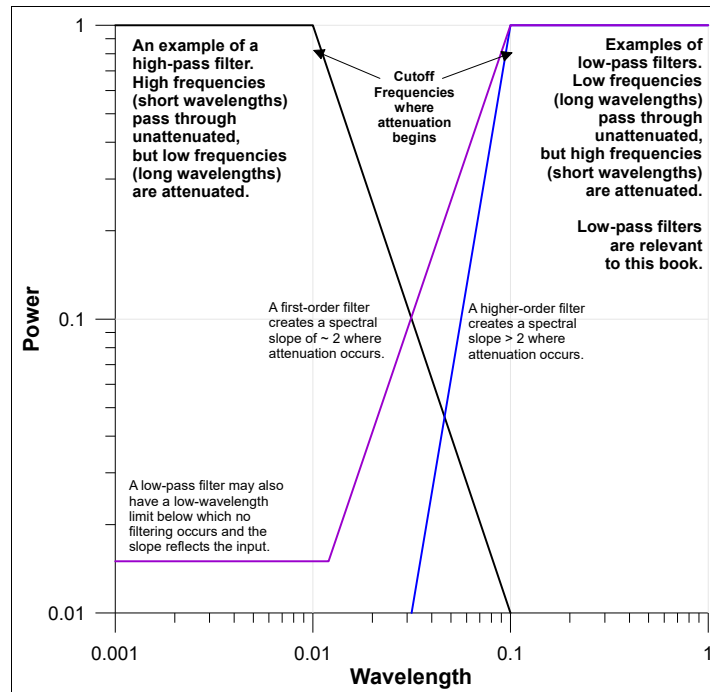


Figure 3-1. Examples of sets of spectral slopes created by non-ideal high-pass and low-pass filters (Figure 4-9 from Morin, 2016).

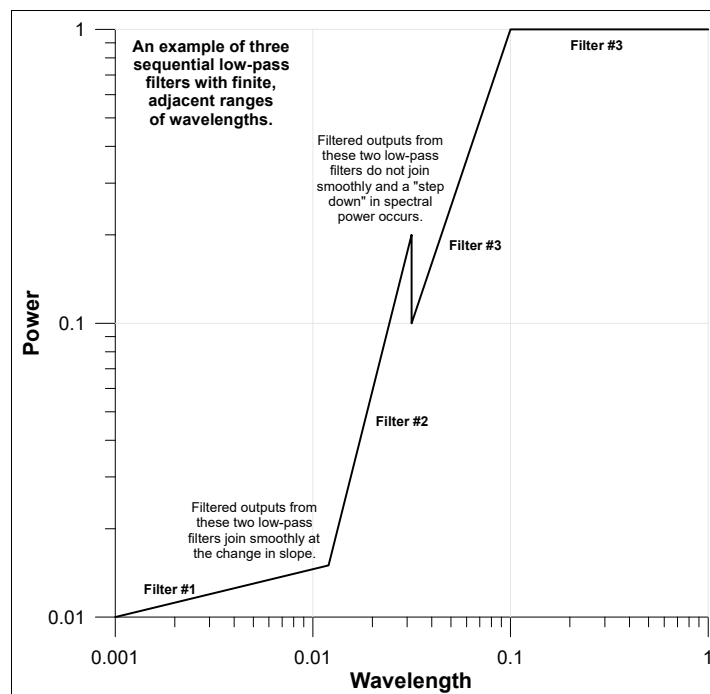


Figure 3-2. An example of spectral slopes created by three wavelength-sequential low-pass filters, showing smooth and abrupt transitions from one filter to another (Figure 4-10 from Morin, 2016).

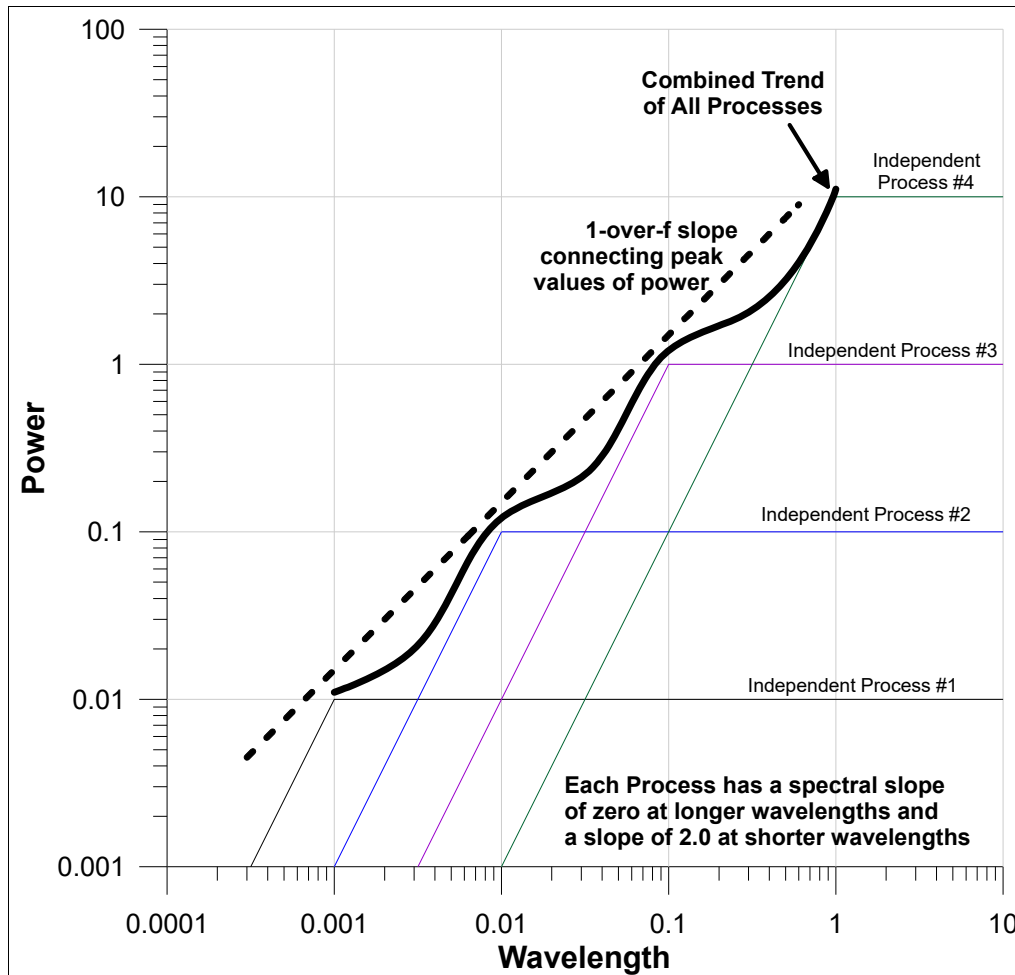


Figure 3-3. Composite spectral power of a system with multiple independent operating processes, with each process producing white noise ($\alpha = 0$) at longer wavelengths and red noise ($\alpha = 2$) at shorter wavelengths; a line connecting the power peaks is 1-over-f ($\alpha = 1$) (Figure 5-6 from Morin, 2016).

The 1-over-f slopes can represent the complex overlap of many, possibly hundreds of, simultaneously operating physical, chemical, and biological processes (Morin, 2016). Attempts to understand and simulate these slopes in non-mining-related rivers and catchments have not been robust and have been generally unsuccessful (e.g., Kirchner, 2009; Kirchner et al., 2000, 2001, and 2004; Kirchner and Neal, 2013). Nevertheless, there is a relatively simple and viable explanation for these slopes in both non-mining catchments and minesite drainages.

3.2 Basic Design and Operation of a Low-Pass Filter

Because minesite components and non-mining catchments filter input “signals” (Figure 2-2) like a first-order low-pass filter (Figures 3-1 and 3-2), it is worthwhile to examine the basic design of one based on electronic components like resistors and capacitors.

Figure 3-4 shows a basic “RC” design from Wikipedia (2020d). A capacitor (“C”) stores energy in an electrical field and generally attempts to maintain a steady voltage across it. One end of the capacitor is attached to ground and the other end is attached to the circuit. The location where the input voltage (V_{in}) is applied is separated from the location of the low-pass-filtered output voltage (V_{out}) by a resistor (“R”) which reduces the flow of current and divides the voltages. This design creates the low-pass filtering shown in Figures 3-1 and 3-2. Higher-order low-pass filters with greater slopes include electrical inductors in their design.

Wikipedia (2020d) provides some simple alternative examples of how a low-pass filter operates. *“This [low-pass] circuit may be understood by considering the time the capacitor needs to charge or discharge through the resistor:*

- *At low frequencies, there is plenty of time for the capacitor to charge up to practically the same voltage as the input voltage.*
- *At high frequencies, the capacitor only has time to charge up a small amount before the input switches direction. The output goes up and down only a small fraction of the amount the input goes up and down. At double the frequency, there's only time for it to charge up half the amount.*

Another way to understand this circuit is through the concept of reactance at a particular frequency:

- *Since direct current (DC) cannot flow through the capacitor, DC input must flow out the path marked V_{out} (analogous to removing the capacitor).*
- *Since alternating current (AC) flows very well through the capacitor, almost as well as it flows through solid wire, AC input flows out through the capacitor, effectively short circuiting to ground (analogous to replacing the capacitor with just a wire).*

The capacitor is not an “on/off” object The capacitor variably acts between these two extremes.”

To be clear, Figure 3-4 represents an electronic circuit known as a first-order low-pass filter that affects voltage. Figures 1-3 to 1-5 display the trend created by such a filter, but these figures do not show spectral analyses of voltage or electrical current. Instead, they show spectral analyses of the physical flow of water and of aqueous ionic concentrations reflecting the operation of a low-pass filter. Section 2 above, specifically Equation 2-2, shows how electrical energy, water flow, and aqueous concentrations can be linked together with variations (including periodicity) in one potentially affecting the others.

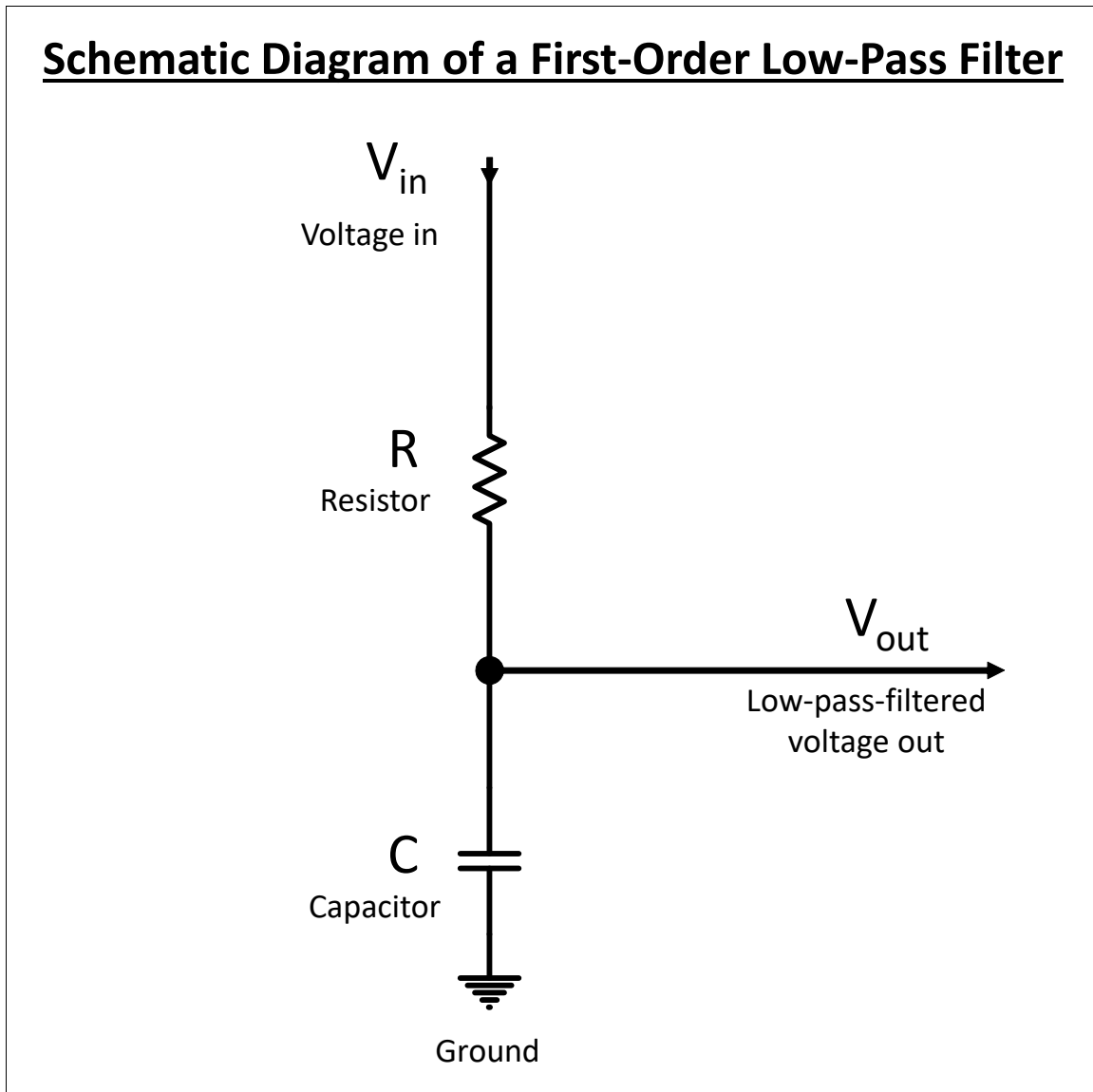


Figure 3-4. A basic schematic diagram of a first-order low-pass RC filter (adapted from Wikipedia, 2020d).

3.3 A Minesite Component Connected to the Earth as a Low-Pass Filter

There is an analogue for Figure 3-4 which literally involves the entire planet.

As explained in the previous MDAG case study (Morin, 2020), atmospheric electricity (AE) involves the generally negative and positive electrical potentials found in the solid earth and the upper atmosphere. The intervening air does not conduct much electricity, although electrical storms occasionally cause local “short circuits”. This turns out to be the basic design of a capacitor (Wikipedia, 2020e). The entire planet along with its atmosphere forms a huge capacitor, with a capacitance of approximately 0.0007 farads (coulombs/volt) and whose one “terminal” is obviously grounded in the earth (e.g., Krider, 2013; TopperLearning.com, 2018). Thus, the bottom of Figure 3-4 is the earth.

Next there is a cyclical input signal with a large range of significant wavelengths entering at V_{in} in the upper part of Figure 3-4. The signal passes through the resistor and creates the low-pass-filtered output signal with a fractal spectral slope at V_{out} . Figure 2-2 already showed this conceptually.

In this analogue, the resistor is a relatively large minesite component, with rock and tailings having finite and relatively large resistances to currents and potentials of electricity, water flow, heat etc. Also, most minesite components are well connected to the earth, because potentially insulating soil and loose rock typically are initially removed to improve geotechnical stability. Therefore, the first-order low-pass filter affecting minesite-drainage flows and chemistry is graphically illustrated in Figures 3-5 to 3-7 based on the electrical low-pass filter of Figure 3-4.

Where a minesite component is not well connected to the earth, such as a collection pond lined with a high resistivity basal liner like a geomembrane, Figures 3-4 to 3-7 may not apply. For example, for the lined collection pond at Minesite 2 (Appendix B5 of Morin, 2016), many spectral slopes are less than 1.0 suggesting the low-pass filter is not filtering those input signals well.

3.4 Calculation of Cutoff Wavelengths

In Figure 3-1 and Figures 1-3 to 1-5, the wavelength at which low-pass filtering becomes significant is often called the “cutoff wavelength” (or “cutoff frequency” = $1/\text{cutoff wavelength}$, Wikipedia, 2020d). Values of cutoff wavelengths can tell us some characteristics of the site -specific and element-specific low-pass filters.

It is important to note that there is a major assumption used here that seems reasonable but has not been confirmed: the cutoff wavelength for flow and chemistry is the same for electrical current and other forms of energy in Equation 2-2. In other words, amplitudes and spectral powers at specific wavelengths may differ among energies, but the cutoff wavelength by the low-pass filter presumably remains the same.

A Minesite Component as a First-Order Low-Pass Filter

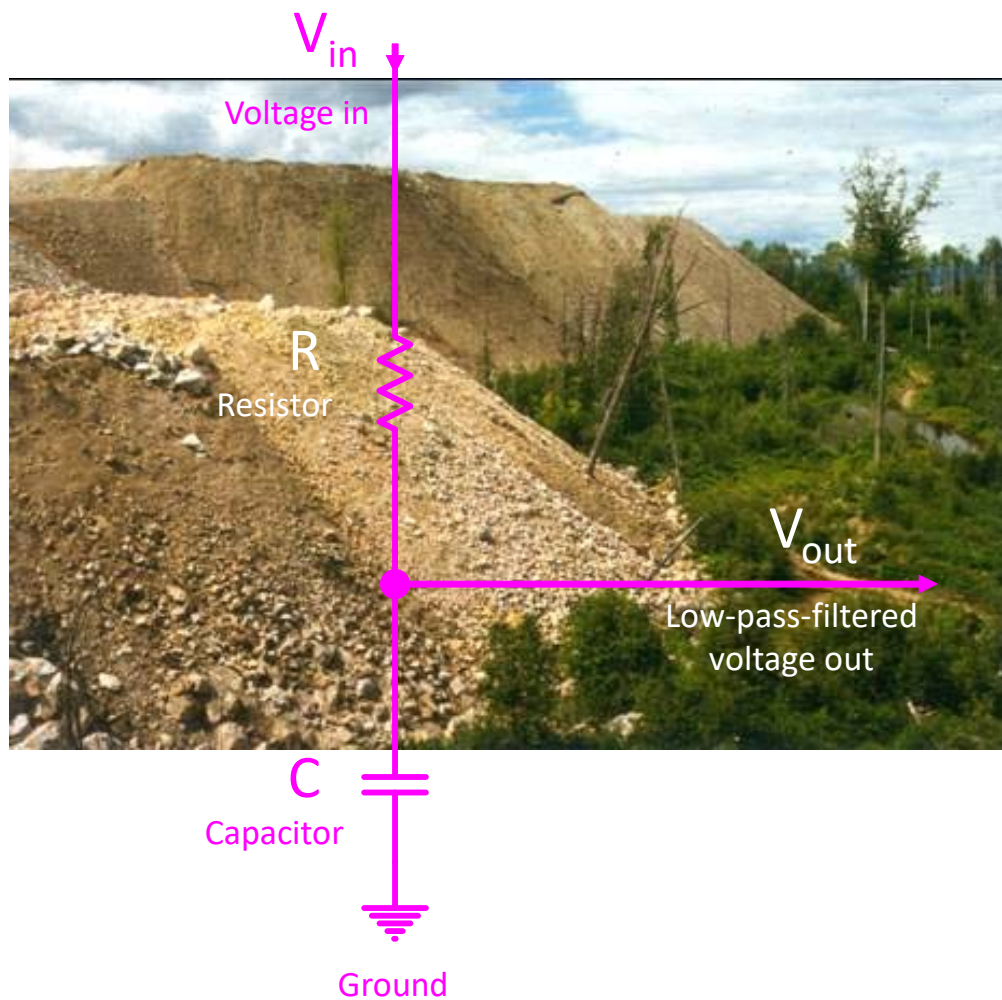


Figure 3-5. An example of a waste-rock pile connected to the earth simulating a low-pass filter on water flow and aqueous chemistry as summarized in Figure 2-2.

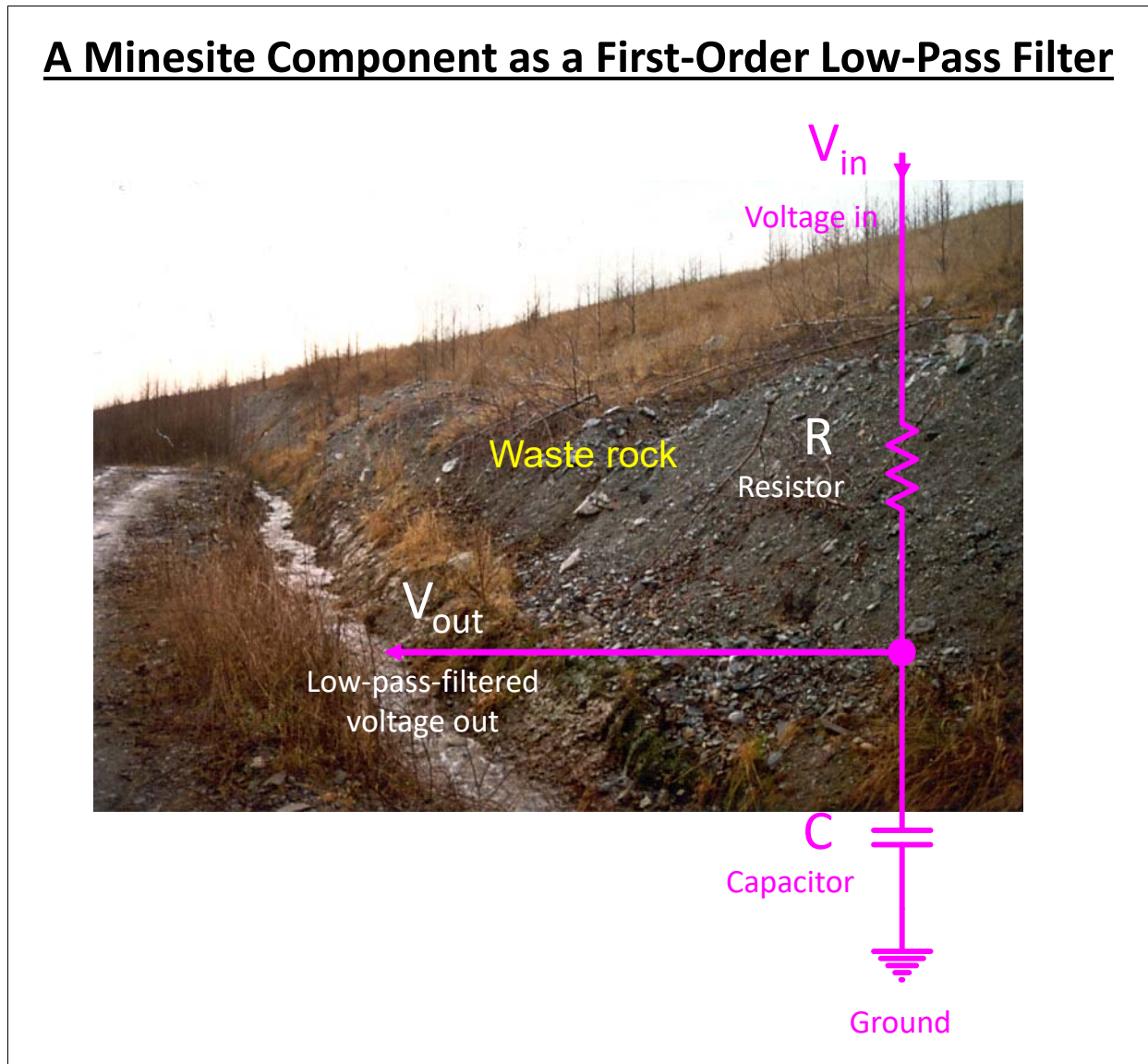


Figure 3-6. An example of a second waste-rock pile connected to the earth simulating a low-pass filter on water flow and aqueous chemistry as summarized in Figure 2-2.

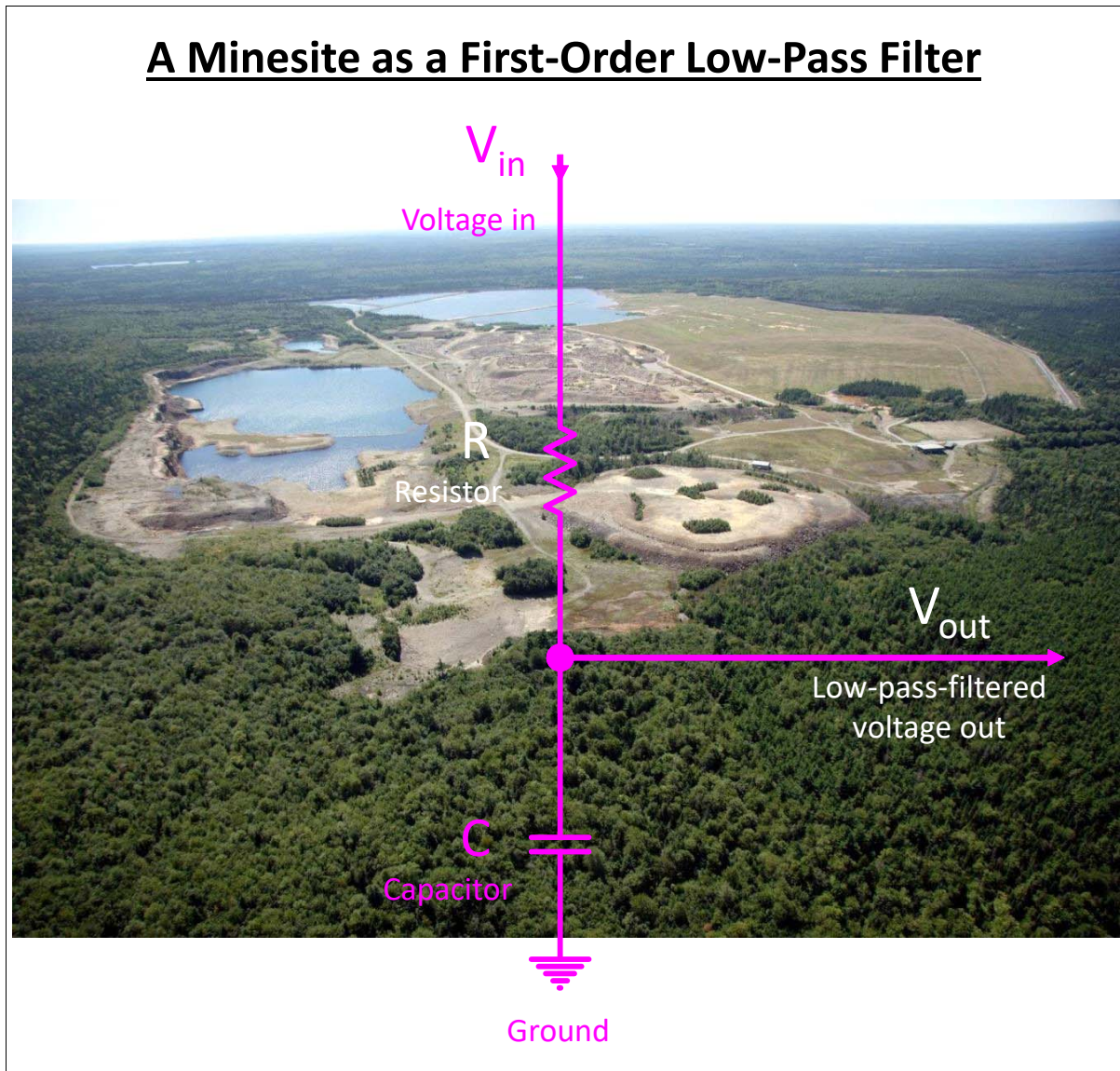


Figure 3-7. An example of an entire minesite connected to the earth simulating a low-pass filter on water flow and aqueous chemistry as summarized in Figure 2-2.

The equation for the cutoff wavelength (e.g., Wikipedia, 2020d) is:

$$w_{c,sec} = 2 \pi R C \quad (\text{Equation 3-1})$$

where:

$w_{c,sec}$ = cutoff wavelength, in seconds

R = resistance, in ohms = 1/siemens = (volts*seconds)/coulomb

C = capacitance, in farads = coulomb/volt

Also, resistance is calculated from:

$$R = r*L/A \quad (\text{Equation 3-2})$$

where:

R = resistance, in ohms = 1/siemens = siemens⁻¹ = (volts*seconds)/coulomb

r = resistivity, in ohm-meter = meter/siemen

L = length of system, in meters

A = cross-sectional area of system in meter²

Equations 3-1 and 3-2 can be combined and the C of the earth added (0.0007 farads) to obtain:

$$w_{c,sec} = 0.00440 * (r*L/A) \quad (\text{Equation 3-3})$$

The cutoff wavelength in seconds can be mathematically adjusted to years for comparison to Figures 1-1 to 1-5:

$$w_{c,yr} = 1.40 \times 10^{-10} * (r*L/A) \quad (\text{Equation 3-4})$$

where:

w_c = cutoff wavelength, in years

3.5 Cutoff Wavelengths Applied to Full-Scale Minesite Components

For cases where only fractal spectral slopes are seen (e.g., Figure 1-1), then (1) the cutoff wavelength is longer than the longest wavelength in the monitoring database or (2) there is no operative low-pass filter. For cases where no low-pass filtering is seen, then (1) the cutoff wavelength is shorter than the shortest wavelength in the monitoring data or (2) there is no operative low-pass filter. Because of this ambiguity, calculations with the cutoff wavelength are best done where it can be seen (e.g., Figures 1-3 to 1-5). Nevertheless, Equations 3-1 to 3-4 offer some greater-than or less-than values for R where the cutoff wavelength appears to be above the maximum monitored wavelength or below the minimum monitored wavelength, respectively.

Within a minesite component, resistivity (r) and resistance (R) can vary temporally and/or spatially (e.g., due to changes in primary and secondary minerals and in water content), and/or vary by energy potential (e.g., electrical vs. pressure vs. temperature), and/or by mechanism (physical, chemical, or biological). As a result, the cutoff wavelengths in the spectral output of flow and chemistry may differ with time, location, potential, and/or mechanism. If most of these each reflect a distinct first-order low-pass filter, then a 1-over-f slope can arise (Figure 1-1 and 3-3). If only one is dominant, then a slope around 2 is expected (Figures 1-4, 1-5, 3-1, and 3-2).

As an example, Figures 1-4 and 1-5 show a $w_{c,yr}$ of approximately 0.5 years ($w_{c,sec} = 1.58 \times 10^7$ seconds) for flow and pH. Based on Equation 3-3, $(r \cdot L/A) = R = 3.94 \times 10^9$ ohms or siemens⁻¹ for the waste-rock pile. If L/A was approximately 1.0, then the resistivity (r) of the total waste-rock component would be 3.94×10^9 ohms·meters or meters/siemens. This value is typical of, for example, relatively dry metamorphic and igneous rocks as found at this site, which is discussed in more detail in Section 3.6.

3.6 Resistance (R) based on Resistivity (r), Length (L), and Cross-Sectional Area (A)

The previous subsection showed how R for a minesite component can be estimated from the cutoff wavelength in the drainage periodogram using Equations 3-1 to 3-4. That was not too difficult. However, estimating r from Equations 3-2 and 3-4 is more complex, partly because r is highly variable by many orders of magnitude and partly because L/A can be ambiguous.

3.6.1 Resistivity (r)

The resistivity of rock, including waste rock and tailings, can vary over more than 15 orders of magnitude! Resistivity varies with combinations of minerals comprising the rock, water content, orientation of mineral layering or heterogeneity (discussed further in Subsection 3.6.2), porosity and degree of fracturing, temperature, pressure, and other factors (e.g., Parkhomenko, 1967; Slichter and Telkes, 1942; McNeill, 1980). Even the reported resistivity of “pure” pyrite spans a factor of 1000.

To simplify this complex scenario, extreme values of resistivity can be considered. Thus:

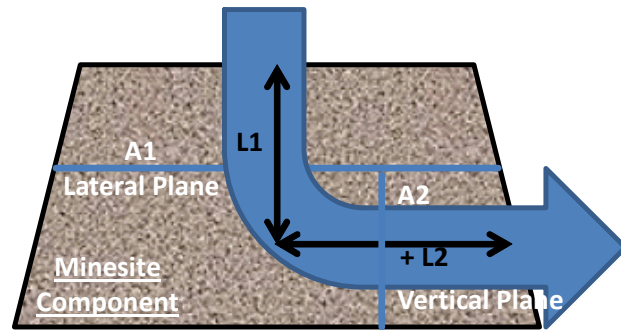
- for dry solid quartz, $r \sim 10^{+12}$ to 10^{+14} meter/siemen (equivalent to the old units of ohm·meter) depending on direction of measurement relative to crystal axes;
- for sulphide minerals and graphite, $r \sim 10^{-5}$ to 1 meter/siemen;
- for groundwater with hundreds to thousands of mg/L typical of minesite drainage, $r \sim 10 - 15$ meter/siemen; and
- for air, $r \sim 10^{+9}$ to 10^{+15} meter/siemen.

3.6.2 Length/Area

In Equations 3-2 to 3-4, L/A is defined as the ratio of system length to system cross-sectional area. If this sounds ambiguous, that is because it certainly is. In some ways, L , A , and r affect each other.

For one of the simplest scenarios, L is the length of the single pathway over which low-pass signal filtering occurs in a minesite component (Figure 2-2) and A is the cross-sectional area of the minesite component perpendicular to that pathway. Simplistically, the input is generally downward for migrating precipitation, turning to lateral flow at the base to the drainage exit (e.g., Figures 3-5 to 3-7). In this case, the first part of L (L_1 in the top diagram of Figure 3-8) is almost the height of the minesite component and A_1 is the lateral area of the component, which means L/A using meters as units would typically be much less than 1.0 by up to orders of magnitude.

Various Configurations for L/A in Minesite Components

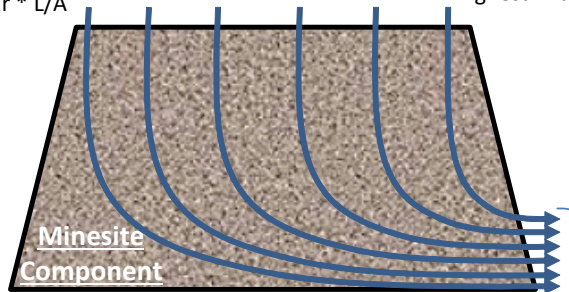


Relatively Simple Scenario of the Entire Minesite Component:

Using units of meters, L/A typically $\ll 1.0$

If a pathway is continuous water, sulphide minerals, graphite, etc., then R for that pathway is: relatively very low $r * L/A$

If a pathway is discontinuous water with intervening minerals, or a series of contacting dry minerals, then R for that pathway (like a series of resistors) is the sum of the individual $(r*L/A)$ with the highest r values dominating the sum



Complex Scenario with Multiple Pathways

Each with a Different Resistance:

Using units of meters, $L/A \gg 1.0$ for each individual long narrow pathway but not the sum

Like resistors in parallel, the combination of pathways has a value of $1/R$ equal to the sum of each reciprocal of $(r*L/A)$ with the lowest values of r dominating the cumulative R

Figure 3-8. Two of many possible configurations of pathway length (L) divided by pathway cross-sectional area (A), with varying resistivities (r), producing a total resistance (R).

When the pathway reaches the bottom, it can turn laterally to the outer edge (“toe”) of the component (top of Figure 3-8). As a result, L_2 is the additional distance, and A_2 involves the vertical cross-sectional area.

This lateral scenario also applies to a common alternative input where lateral water movement, such as a creek or groundwater system, enters through a vertical cross-section on one side and exits at another side. In these cases, L/A using meters would again typically be much less than 1.0 by up to orders of magnitude.

Therefore, in this simple scenario, L/A would be much less than 1.0. In turn, Equation 3-2 shows that $r \gg R$. As a result, the numerical example in Section 3.5 means $r \gg 3.94 \times 10^9$ meters/siemens, which would be typical of relatively dry quartz-rich rock.

On the other hand, a complex scenario is that minesite components contain heterogeneous mixtures of countless particles of various wet and dry minerals with different resistivities. Each mineral grain with a certain r is in contact with other mineral grains and water with other r values parallel to a particular pathway.

In this case, L is the length of each pathway parallel to the flowpath (bottom of Figure 3-8), and A is the perpendicular cross-sectional of each pathway. In this scenario using meters, L/A would be much greater than 1.0 by up to orders of magnitude for each pathway if there are many pathways.

This complex scenario can be envisioned as a long string of electrical resistors (“ n ” as the number of resistors) connected in series. Thus, the total resistance along the pathway is:

$$R = r_1 * L_1 / A_1 + r_2 * L_2 / A_2 + \dots + r_n * L_n / A_n \quad (\text{Equation 3-5})$$

If each L/A is identical, Equation 3-5 can be reduced to:

$$R = L/A * (r_1 + r_2 + \dots + r_n) \quad (\text{Equation 3-6})$$

In these equations for each pathway, the highest values of r can dominate the summation and dominate pathway R , opposite to Equations 3-7 and 3-8 below.

In addition to each pathway as a series of resistors, the pathways can come together at the exit of the low-pass filter. This resembles the common view of full-scale waste-rock piles containing a wide range of permeable pathways (“channels”) for water (e.g., Morin et al., 1991; Smith et al., 1995; Li, 1999).

For example, one pathway might be continuously connected throughout by water with $r \sim 10 - 15$ meters/siemen, multiplied by its L/A . Next to that there might be a continuous pathway of dry quartz-rich rock with $r \sim 10^{+12}$ to 10^{+14} meters/siemen, multiplied by its L/A . And next to that might be a continuous pathway of sulphide minerals with $r \sim 10^{-5}$ to 1 meters/siemen multiplied by its L/A . This can be reduced to a set of electrical resistors (“ n ” in number) in parallel. In this case:

$$1/R = A_1 / (r_1 * L_1) + A_2 / (r_2 * L_2) + \dots + A_n / (r_n * L_n) \quad (\text{Equation 3-7})$$

If each A/L is identical, Equation 3-7 can be reduced to:

$$1/R = A/L * (1/r_1 + 1/r_2 + \dots + 1/r_n) \quad (\text{Equation 3-8})$$

In these equations, the lowest values of r can dominate the reciprocal summation and dominate the value of R , opposite to Equations 3-5 and 3-6 above.

Parkhomenko (1967) provides similar and alternative equations to Equations 3-5 to 3-8 depending on whether the pathway is parallel to, or transverse to, “bedding planes” and fracture planes.

This complex interplay of r , L , and A in Equation 3-2 will be further explored in a later MDAG case study. For example, in minesite components with high water contents or abundant sulphide minerals, R and the corresponding cutoff wavelength can be very short and perhaps not detected, and thus monitoring data would show mostly unfiltered and/or zero spectral slopes. For components with high R , such as due to dryness or abundant resistive minerals like quartz, R the corresponding cutoff wavelength can be very long and may not be detected. Thus, monitoring data would mostly show only a low-pass spectral slope around 2, or the more complex $1\text{-over-}f$ slope if several slopes around 2 superimpose (Figure 3-3).

As a final note, it is important to note in Equation 3-3 that if $(R = r*L/A)$ is very small, such as 1.0, the cutoff wavelength will be much less than one second and thus may not be detectable. The periodicity will not be affected and filtered by such a low-pass filter. For example, if the pathway contains continuous water with a low r , then L/A will have to be large to detect the cutoff wavelength. Therefore, the size of the minesite component and Figure 3-8 play major roles in whether cutoff wavelengths are detected at all within the spectral analysis of the monitoring database.

4. Conclusion

Two questions were asked in the Introduction of this MDAG case study. Here are the questions again, with brief answers based on the preceding pages.

- 1) What mechanisms and processes can introduce periodicity into minesite-drainage chemistry and flow? In other words, why do flow and chemistry “pulsate” at many wavelengths? This effect is more “ubiquitous” in non-mining catchments.

Answer: Based on Equation 2-2 reflecting various forms of energy and their interactions, the oscillation in any energy term can produce periodicity in one or more other energy terms. This includes the 11-yr sunspot cycle, the 2.38-yr quasi-biennial oscillation in tropical zonal winds, rotational gravity waves, and various tides driven by motions of the moon and sun relative to the earth. These appear to be linked to flow and aqueous concentrations primarily through oscillations in large-scale electrical fields based on electrodynamics, electrochemistry, etc. (Morin, 2020).

- 2) How do these mechanisms apparently “self organize” so that their amplitudes and spectral powers generally decrease with decreasing wavelength, often forming fractal spectral slopes? This includes the very common yet mysterious “1-over-f” with a spectral fractal slope of 1.0, which should not appear and persist according to mathematics.

Answer: Minesite components are open systems in the surficial environment well grounded to the earth that behaves like a capacitor. Thus, relatively large minesite components can act as first-order low-pass filters. These filters cause the spectral powers of various periodicities entering them to (1) decrease along a fractal slope of 2 at wavelengths shorter than the “cutoff wavelength” and (2) remain unfiltered at longer wavelengths. When several mechanisms are simultaneously acting as low-pass filters, 1-over-f slopes can appear.

If these answers withstand scrutiny, then fractal spectral slopes including 1-over-f slopes should be common in drainages from relatively large minesite components, and they are, and thus not mysterious and unexplainable.

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