# MDAG.com Internet Case Study 56

# MDAG-com Case Study 56 - Examples of Wavelet Transforms of 1-over-f and Other Temporal Spectral Slopes in Minesite Drainage

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www.mdag.com/case_studies/cs56.html

## TABLE OF CONTENTS

## Abstract

In past MDAG work, least-squares spectral analyses were conducted on dozens of high-frequency and long-term time series from full-scale minesite monitoring. Spectral analysis, as a generalization of the Fourier Transform, reveals the significant wavelengths (e.g., daily, weekly, monthly, annually) at which various physical, geochemical, and biological processes operate.

The power spectrum, or periodogram, from spectral analysis is a type of integrated summary of significant wavelengths over the entire monitoring period. For example, over a ten-year monitoring period, year-by-year changes would not necessarily be revealed by the power spectrum. In contrast, the wavelet transform can provide year-by-year information on significant wavelengths.

This MDAG case study presents examples of time series and power spectra for pH, zinc, copper, and flow at the outflow from a pit, backfilled with reactive waste rock, and overlain by a till-soil cover. These examples had log-log spectral-power slopes ($\alpha$) ranging from 0 (random or chaotic), through 1 (1-over-f), to 2 (random walk).

Wavelet coefficients were used to evaluate previous interpretations and to obtain new insights on these time series and power spectra. As an example, aqueous pH and zinc were found to respond to typical sulphide-oxidation and metal-leaching paradigms, whereas aqueous copper did not. Also, the seasonal variability of outflow in spring increased significantly after the installation of the overlying till-soil cover.

## 1. Introduction

Morin (2016) used least-squares spectral analysis in the wavelength-frequency domain to identify the major wavelengths of periodicities in three case studies of full-scale minesite drainage. The major wavelengths reflect underlying cyclical processes or fluxes, caused by myriad physical, chemical, and biological factors, in and around the "open systems" of minesite components.

Due to the unevenly spaced and irregular monitoring data inherent in most environmental databases, Morin (2016) used the Lomb-Scargle algorithm for spectral analysis. This algorithm was available online at the NASA Exoplanet Archive website (Akeson et al., 2013).

The three mining case studies yielded many straight-line spectral slopes (α) on log-log plots. These slopes included random (or chaotic, α~0), random walk (α~2), and 1-over-f (α~1). One-over-f slopes are a trendy topic[1], with much research devoted to explaining their existence in so many sciences. The most popular explanation is self-organized criticality (Bak, 1996), where all contributing processes and fluxes "self-organize" across a range of scales.

The results of spectral analysis, as a generalization of the Fourier Transform, can be thought of as a summation or integration of spectral powers across the entire monitoring interval. For example, if ten years of monitoring data are examined by spectral analysis, the results reflect all ten years. However, over ten years, the following two scenarios can produce similar power spectra:

1) five years of short-wavelength periodicity followed by five years of long-wavelength periodicity, and
2) ten years of combined short- and long-wavelength periodicity.

To distinguish between these and other complexities, it would be nice to "break" the monitoring period into shorter "windows", and check for periodicities in those shorter periods. That is what "wavelet transforms" do (Wikipedia, 2018), using cyclical mathematical equations applied to sequential periods of time and to sequential wavelengths.

It is not the intent here to explain the application of wavelets to minesite drainage in detail; that will come in a later document. For those interested in details now, this MDAG case study used the continuous Morlet wavelet, with a method (Keitt, 2008) for handling the unevenly spaced and irregular minesite-monitoring data that invalidates most wavelet software. Wavelet coefficients were obtained using Package "mvcwt" in the R Language Version 3.4.4 (R Core Team, 2016), with the Rstudio 1.1.447 user interface (Rstudio Team, 2015). Package "mvcwt" does not provide the typical absolute values for the wavelet coefficients, and thus phase information (positive or negative) is included in these coefficients. Results were imported into Surfer 15 (Golden Software, 2018) for plotting, using a base-10 logarithm for Scale rather than the more traditional base-2 logarithm.

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[1] http://www.mdag.com/1-over-f-slopes.html

## 2. Examples of Minesite Drainage with Unevenly Spaced Data

This MDAG case study focusses on aqueous pH, zinc, and copper, as well as physical water flow, at a particular monitoring location in Case Study 2 of Morin (2016). This location is the outflow from the Southern Tail Pit (STP), backfilled with reactive waste rock, and covered with a till-soil cover before mine closure in Mine Year 14.

This particular location was discussed in detail in Section 7.4.1 and Appendices B2 and B3 of Morin (2016). The spectral slopes for the aqueous parameters (~600 samples from Mine Year 5 to 30) were:

- α~0 (random white noise) up to a wavelength of ~ one month (~0.1 year), and,
- at higher wavelengths, α~1 (1-over-f) for zinc and copper and α~1.4 for pH.

For flow (1544 measurements from Mine Year 10 to 30):

- α was irregular below a wavelength of ~0.02 years (~ one week), and
- α~2 (random walk) up to a wavelength of a half year.

These spectral slopes are shown in the top right of Figures 3-1 and 3-4 below.

Wavelet techniques, just like spectral analysis (Morin, 2016) and other statistical techniques (e.g., Section 1.5 of Morin, 2018), typically expect input data to have absolutely equal and consistent spacing. That is highly unlikely for environmental databases that suffer from issues like occasional equipment failure and loss of data or calibration.

For aqueous pH, zinc, and copper, in outflow from the Southern Tail Pit, the sampling frequency has been uneven and erratic, between and within decades (Figure 2-1). Measurements of flow were also uneven (Figure 2-2). This precluded the usage of most wavelet software, and instead required a special wavelet technique as summarized at the end of Section 1 above.

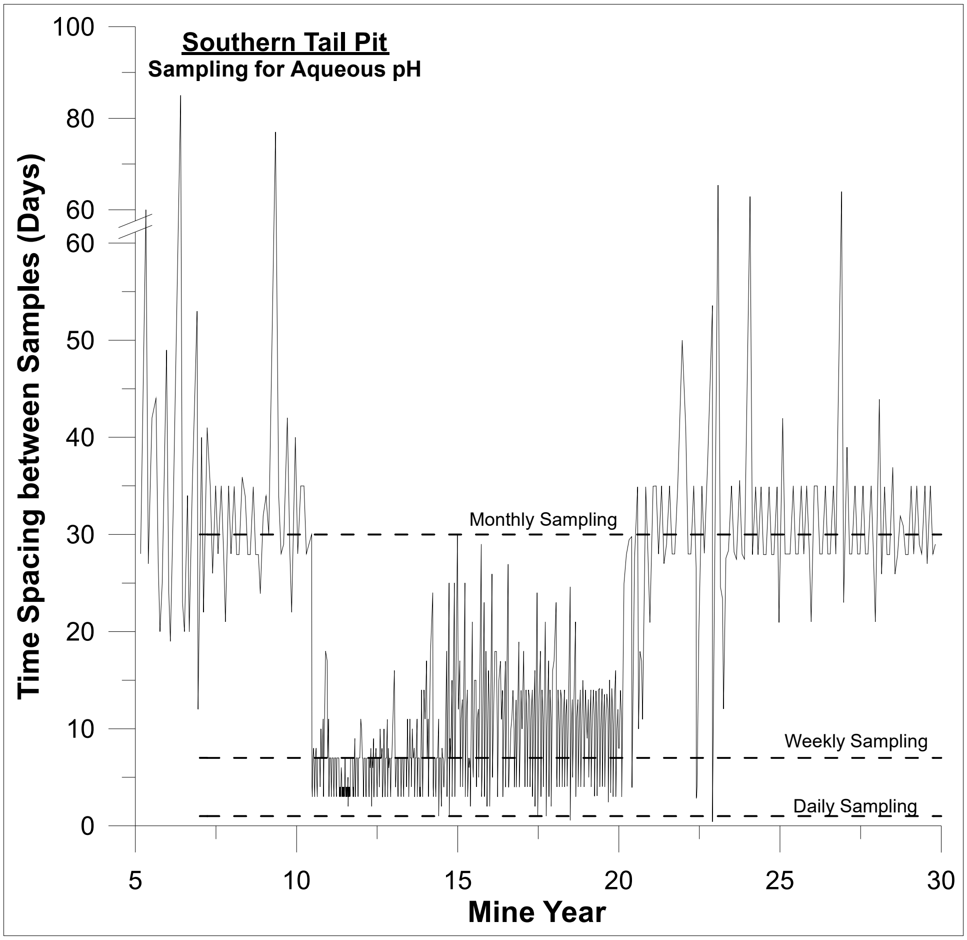

**Figure 2-1. The number of days between consecutive water samples for pH, zinc, and copper in outflow from the backfilled Southern Tail Pit; note: even sampling for most wavelet software would appear as a 100% consistently horizontal line in this figure.**

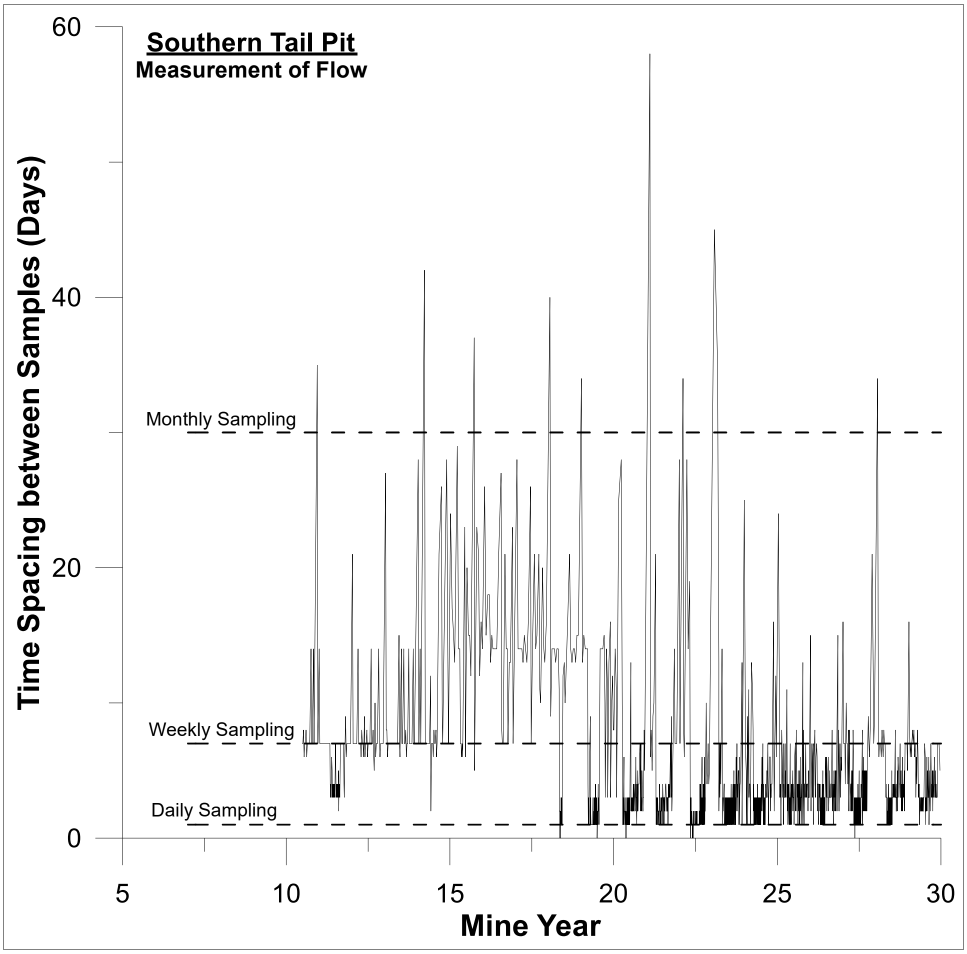

**Figure 2-2.** **The number of days between consecutive measurements of outflow from the backfilled Southern Tail Pit; note: even sampling for most wavelet software would appear as a 100% consistently horizontal line in this figure.**

## 3. Wavelet Transforms for These Examples of Minesite Drainage

In the outflow from the backfilled Southern Tail Pit (STP), the temporal trends and power spectra for aqueous pH, zinc, copper, and flow are shown on the top left and top right sides of Figures 3-1 through 3-4, respectively. Sharp changes in values over short periods of time can distort power spectra. Therefore, the power spectra in Figures 3-1 to 3-3 do not include an early period, before Mine Year 10, when temporal trends of pH and aqueous concentrations shifted strongly.

Wavelets are more capable of handling sharp changes and minimizing overall distortion in the results. However, wavelets are subject to "edge effects", where trends close to the start time (left axis), end time (right axis), and longest wavelengths (top axis) are not reliable. Since the sharp changes in the STP geochemical parameters occurred in the initial monitoring years, the wavelet results for the first few years must be used with caution.

The wavelet coefficients in this MDAG case study (e.g., STP pH at the bottom of Figure 3-1) are either in positive phase in shades of green, or in negative phase in shades of red. The intensity of the colour reflects the amplitude. A "positive phase" is like the upper part of a cycle (like the arch in a sine curve), whereas the "negative phase" is like the lower part of a cycle (like the dip in a sine curve).

### 3.1 Aqueous pH in Outflow from the Backfilled Southern Tail Pit

For STP pH (bottom of Figure 3-1), the most obvious wavelet pattern is the shift from random or chaotic at a log10 Scale (the wavelength) of about -1.1 to -1.2 years, which corresponds to 0.06-0.08 years or almost one month. Above this transition, the wavelet coefficients are more orderly and patterned, appearing as vertical bifurcating "fingers", with less orderly but still some distinctive patterns below the transition. If self-organized criticality (Section 1) applies to STP pH (as well as the other STP parameters discussed below), then for STP pH the wavelength above which self-organization occurs is about one month. This is consistent with its power spectrum (top right of Figure 3-1).

Figure 2-1 shows that weekly monitoring was generally available during Mine Years 10-20, with roughly monthly sampling before and after this interval. Thus, the transition wavelength to organized periodicity at about one month cannot be reliably estimated from monthly sampling, and is thus an extrapolation in the wavelet plot of Figure 3-1 outside Mine Years 10-20.

In contrast, the primarily weekly sampling between Mine Years 10-20 would allow a more accurate estimate of the transition wavelength, which does confirm the organizational transition at about one month. Furthermore, because monitoring data are more frequent than one month in this interval, some significant periodicity can be seen at shorter wavelengths. For example, between Mine Years 10 and 13, just before mine closure and till-cover completion, stronger periodicity (richer colours are greater amplitudes) occurs at log10 Scale -1.6 to -2.1, which corresponds to 3 to 9 days. The time series (top left of Figure 3-1) confirms that extreme values were more frequently encountered just before mine closure than after.

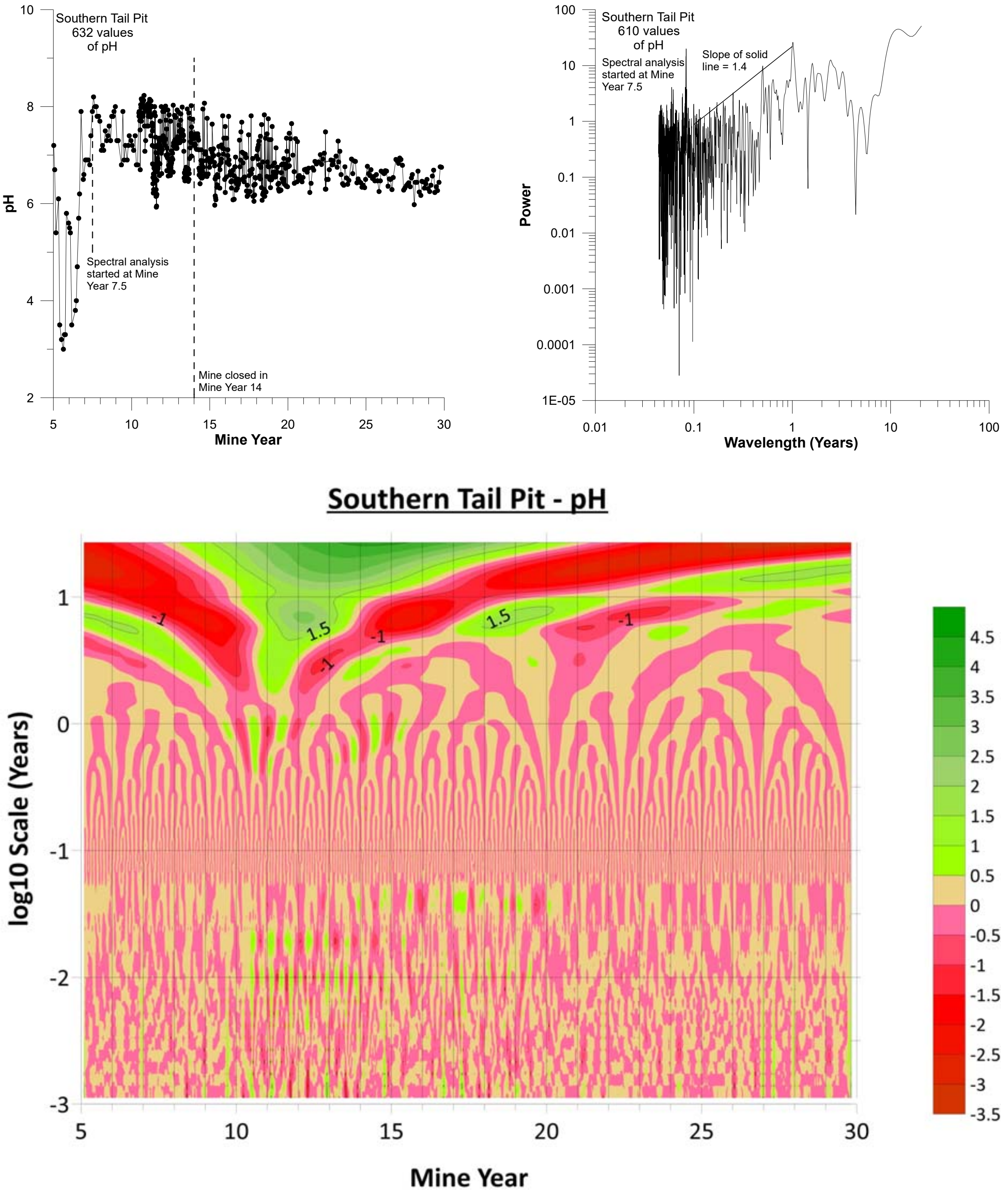

**Figure 3-1. Aqueous pH in the outflow from the backfilled Southern Tail Pit: (top left) time series, (top right) power spectrum, (bottom) wavelet coefficients; note: edge effects of the wavelet transform greatly lower the reliability of values near the left, right, and top axes of the wavelet plot.**

After closure and through Mine Year 20, the stronger periodicity for STP pH appears to shift to a longer wavelength around log10 -1.3 to -1.4 (15 to 18 days) as extreme values are less frequently encountered. The strength of periodicity then weakens (washed-out colours are weaker amplitudes) after Mine Year 20, which is consistent with the lower annual variability seem in the time series after Mine Year 20.

At log10 Scales above 0 (more than one year), periodicity for STP pH remains weak until roughly log10 Scale of +0.9 to +1.0, which corresponds to wavelengths of 8 to 10 years. In agreement, the power spectrum (top right of Figure 3-1) also shows weak power between roughly 1 and 8 years, then increasing spectral power at longer wavelengths perhaps representing long-term trends. Through Mine Year 30, the wavelet plot shows ongoing, but weakening, periodicity for STP pH at a wavelength of 10 years (follow the horizontal log10 Scale line of +1 in the bottom plot of Figure 3-1).

## 3.2 Aqueous Zinc in Outflow from the Backfilled Southern Tail Pit

Aqueous zinc displays a spectral 1-over-f slope (top right of Figure 3-2) down to a wavelength of about 0.1 years (about one month), as does aqueous copper (right top of Figure 3-3). However, after Mine Year 10, zinc concentrations generally increase (top left of Figure 3-2), whereas copper concentrations generally decrease (top left of Figure 3-3). This is discussed further in Section 3.3.

Like pH (bottom of Figure 3-1), zinc has the same scale transition wavelength to organization (vertical “fingers”) starting at almost one month (log10 Scale of -1.1 to -1.2, bottom of Figure 3-2). This corresponds to the minimum extent of the spectral 1-over-f slope.

The wavelet plot for zinc is similar in many ways to that of pH, except at longer wavelengths. This makes sense. At shorter wavelengths with weaker coefficient values, the arithmetic values of the coefficients in these wavelet plots cannot clearly reveal the differences between $\alpha$=1.4 for pH and $\alpha$=1.0 for zinc. Showing these “finer” differences is possible using logarithmic values, but logarithms of negative wavelet coefficients are undefined.

At longer wavelengths between log10 Scale of 0 and +1 (between 1 and 10 years), there are some noticeable differences in the wavelet plots of pH (bottom of Figure 3-1) and zinc (bottom of Figure 3-2). This was expected from the power spectra that showed the strength of the pH periodicity did not increase between 1 and 10 years, whereas the strength of zinc periodicity did increase log-linearly through its 1-over-f slope.

As a result, between wavelengths of 1 and 10 years, zinc shows more persistent, strong periodicity across the monitoring period. For example, at the wavelength of 10 years (follow the horizontal log10 Scale line of +1 in the bottom plots of Figure 3-1 and 3-2), the periodicity of zinc remains strong with a positive phase in the last years, compared with the fading strength for pH.

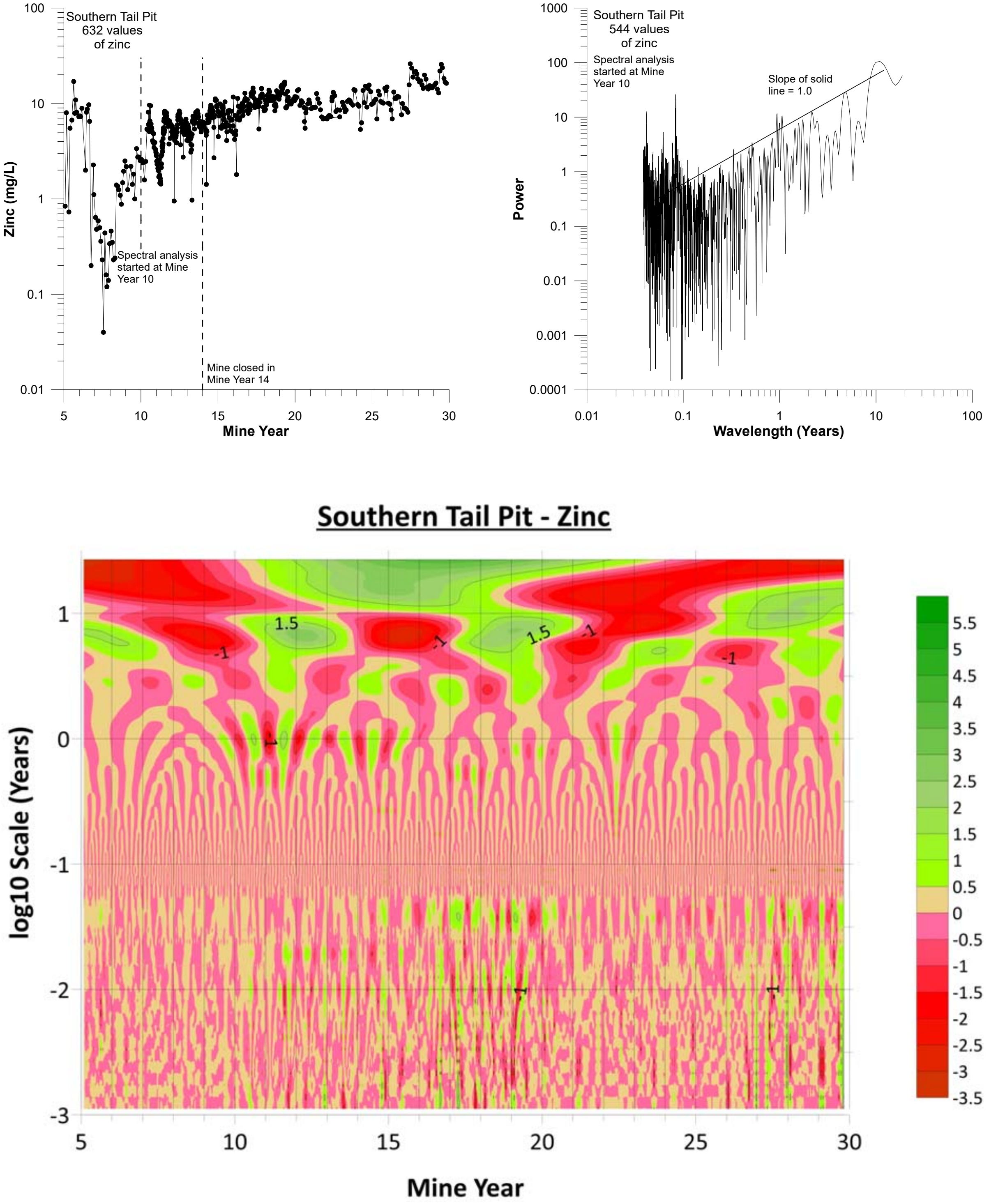

Figure 3-2. Aqueous zinc in the outflow from the backfilled Southern Tail Pit: (top left) time series, (top right) power spectrum, (bottom) wavelet coefficients; note: edge effects of the wavelet transform greatly lower the reliability of values near the left, right, and top axes of the wavelet plot.

## 3.3 Aqueous Copper in Outflow from the Backfilled Southern Tail Pit

Aqueous copper displays a spectral 1-over-f slope (top right of Figure 3-3) down to a wavelength around 0.08 years (~1 month), like zinc (top right of Figure 3-2). However, through Mine Year 30, copper concentrations generally decrease with a late upward cycle (top left of Figure 3-3), while zinc generally increases and pH becomes more acidic (top left of Figures 3-1 and 3-2). Thus, copper is responding to different processes than those associated with typical sulphide oxidation and metal leaching affecting pH and zinc.

In agreement, the wavelet plot for copper (bottom of Figure 3-3) is notably different in some ways from the two similar ones for pH and zinc (bottom of Figures 3-1 and 3-2). An overall difference is that the colours are more "washed out" for copper, indicating the amplitudes of the significant wavelengths are overall weaker. However, there is virtually no difference in the scale transition wavelength to organization (vertical "fingers") starting at almost one month (log10 Scale of -1.1 to -1.2, bottom of Figure 3-2).

After closure in Mine Year 14, copper concentrations generally decrease with few "upswings". This is consistent with the overall weak or negative wavelet coefficients (bottom of Figure 3-3). A notable upswing begins at Mine Year 27, but this does not appear in the wavelet plot, presumably because it is masked by the edge effect of the wavelet transform.

## 3.4 Flow Rate of Outflow from the Backfilled Southern Tail Pit

Unlike the aqueous geochemistry based on samples collected starting Mine Year 5, flow was not recorded until Mine Year 10 (top left of Figure 3-4). Thus, there are five fewer years of monitoring for flow at the beginning.

A major difference between the wavelet plot for flow (bottom of Figure 3-4) and the wavelet plots for the aqueous geochemistry (bottom of Figures 3-1 to 3-3) is the scale transition wavelength to organization (vertical "fingers"). For flow, organized periodicity arises at and above log10 Scale of -1.6 to -1.7 (about 7-9 days, or about one week), compared to almost one month for the geochemical parameters. This may be related to the typical measurements of flow on a daily to weekly basis (Figure 2-2), compared to typical sampling of weekly to monthly for aqueous geochemistry (Figure 2-1), which suggests an artifact.

However, it is consistent with the spectral random-walk slope ($\alpha$=2, top right of Figure 3-4) starting at a wavelength around 0.03 years (11 days) up to 0.5 years. Therefore, the scale transition wavelength may not be a mathematical artifact, but some skepticism is still warranted here.

The time series for flow (top left of Figure 3-4) shows strong annual variations in flow. As a result, the wavelet plot for flow (bottom of Figure 3-4) shows strong amplitudes (richer colours) extending down almost to a wavelength of 0.1 years (~monthly). However, these strong amplitudes at annual and sub-annual wavelengths do not appear until Year 18, four years after the mine closed and the overlying till cover was completed. This may reflect stabilization of the overlying till-soil cover and its transmission of highly variable, short-term infiltration in spring months as snow is melting at this minesite.

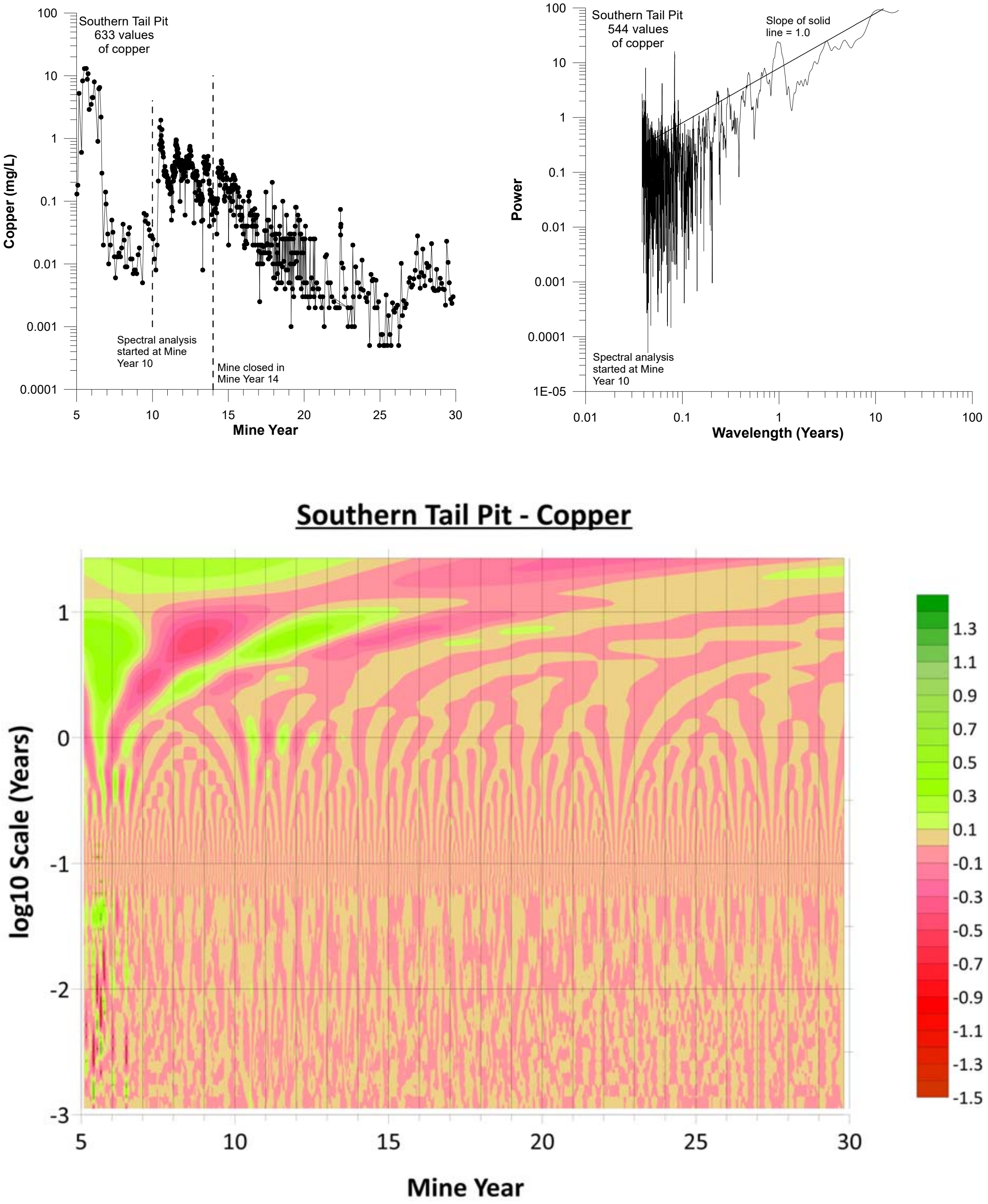

**Figure 3-3. Aqueous copper in the outflow from the backfilled Southern Tail Pit: (top left) time series, (top right) power spectrum, (bottom) wavelet coefficients; note: edge effects of the wavelet transform greatly lower the reliability of values near the left, right, and top axes of the wavelet plot.**

At wavelengths greater than about 2 years (log10 Scale ~ 0.3), the colour intensity is generally weaker than the brighter sub-annual peaks starting Mine Year 18 (bottom of Figure 3-4). This is consistent with the decrease in spectral power after 0.5 years (top right of Figure 3-4). Thus, although there may be longer-term trends in flow, such as around 10 years (log10 Scale of +1), the amplitudes are not strong, and are thus not readily apparent in the time series (top left of Figure 3-4).

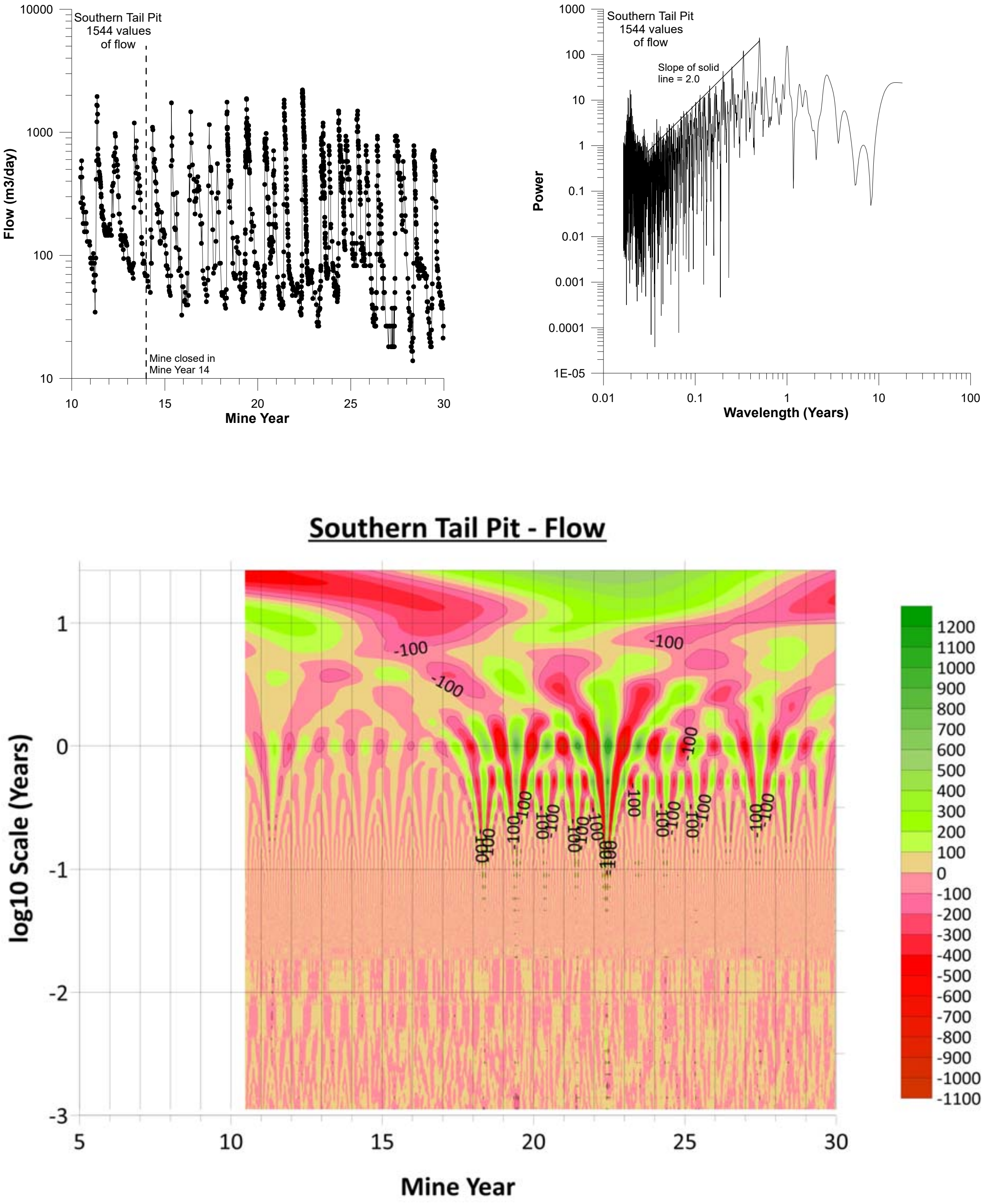

**Figure 3-4.** **Flow rate in the outflow from the backfilled Southern Tail Pit: (top left) time series, (top right) power spectrum, (bottom) wavelet coefficients; note: edge effects of the wavelet transform greatly lower the reliability of values near the left, right, and top axes of the wavelet plot.**

## 4. Conclusion

This MDAG case study has presented examples of wavelet transforms of minesite-drainage time series for pH, zinc, copper, and flow. These examples had log-log spectral-power slopes (α) ranging from 0 (random or chaotic), through 1 (1-over-f), to 2 (random walk).

The time-windowed wavelet plots were interpreted in light of the corresponding time series and time-integrated spectral-power plots, to evaluate previous interpretations and to obtain new insights. For example, aqueous pH and zinc respond to typical sulphide-oxidation and metal-leaching paradigms, whereas aqueous copper does not. Also, the spring variability of outflow increased significantly after the installation of the overlying till-soil cover.

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